

(12) INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

(19) World Intellectual Property Organization  
International Bureau



(43) International Publication Date  
18 April 2002 (18.04.2002)

PCT

(10) International Publication Number  
**WO 02/31660 A2**

(51) International Patent Classification<sup>7</sup>: **G06F 12/02**

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(21) International Application Number: PCT/GB01/04506

(81) Designated States (*national*): AE, AG, AL, AM, AT, AU, AZ, BA, BB, BG, BR, BY, BZ, CA, CH, CN, CO, CR, CU, CZ, DE, DK, DM, DZ, EC, EE, ES, FI, GB, GD, GE, GH, GM, HR, HU, ID, IL, IN, IS, JP, KE, KG, KP, KR, KZ, LC, LK, LR, LS, LT, LU, LV, MA, MD, MG, MK, MN, MW, MX, MZ, NO, NZ, PH, PL, PT, RO, RU, SD, SE, SG, SI, SK, SL, TJ, TM, TR, TT, TZ, UA, UG, US, UZ, VN, YU, ZA, ZW.

(22) International Filing Date: 10 October 2001 (10.10.2001)

(84) Designated States (*regional*): ARIPO patent (GH, GM, KE, LS, MW, MZ, SD, SL, SZ, TZ, UG, ZW), Eurasian patent (AM, AZ, BY, KG, KZ, MD, RU, TJ, TM), European patent (AT, BE, CH, CY, DE, DK, ES, FI, FR, GB, GR, IE, IT, LU, MC, NL, PT, SE, TR), OAPI patent (BF, BJ, CF, CG, CI, CM, GA, GN, GQ, GW, ML, MR, NE, SN, TD, TG).

(25) Filing Language: English

(26) Publication Language: English

(30) Priority Data:  
0024927.6 11 October 2000 (11.10.2000) GB

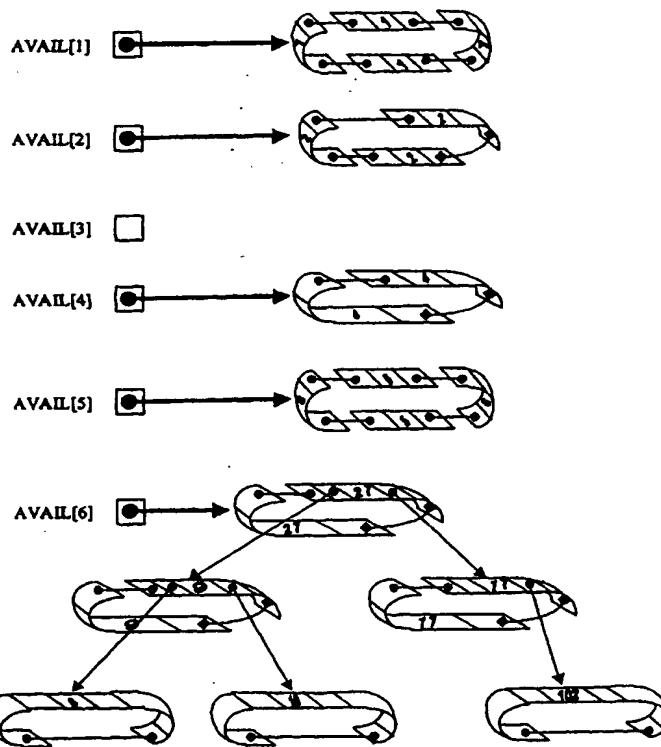
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*[Continued on next page]*

(54) Title: A DATA STRUCTURE, MEMORY ALLOCATOR AND MEMORY MANAGEMENT SYSTEM



(57) Abstract: We present a Best Fit allocator for dynamic memory management. Portions of the memory that are presently unused are call free cells, and each free cell has a size. The allocator uses a bitmap which, for each number of predetermined sizes, indicates whether free memory cells of that size exist. It also employs a second data array with an entry for each of the predetermined cell sizes. When one or more free cells of a given size exist, the corresponding entry of the data array is a pointer to one of those free cells. The free cells themselves contain pointers to other free cells of the same size, or to free cells that are slightly smaller or larger. The allocator is scalable, in that the worst-case behaviour is independent of the size of the heap, and is independent of the number of free cells and of the number of cells already in use for memory storage. It is also incremental and non-disruptive, in that each memory operation (including splitting and coalescing of free cells) is guaranteed to complete within a small bounded time. We also present a novel collector and a priority queuing mechanism that operate on principles similar to those of the allocator.

**WO 02/31660 A2**

**WO 02/31660 A2**



**Published:**

— without international search report and to be republished upon receipt of that report

*For two-letter codes and other abbreviations, refer to the "Guidance Notes on Codes and Abbreviations" appearing at the beginning of each regular issue of the PCT Gazette.*

A Data Structure, Memory Allocator and Memory Management System

Field of the invention

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The present invention relates to storage of data in a data storage space supported by one or more data storage devices, and in particular to a data structure for storing data within the data storage space. The invention 10 further relates to a data storage method using the data structure, to a memory freeing method using the data structure, to a data sorting method using the data structure, and to apparatus arranged to perform any of the above methods.

15

Background of the invention

Many of the tasks performed by modern computer systems require the storage of large quantities of data using a wide variety of data storage devices. In many of these 20 tasks the total quantity of the data to be stored is not known in advance, and neither is the size of the individual packets of data which are sent for storage. Furthermore, the time for which data must be stored varies widely according to the nature of the 25 computational task and the data itself.

For example, consider a computer system which is used for image enhancement. The computer system may contain a bank of original stored images of varying sizes which are held 30 in a relatively long term storage system until it is time for one of those images to be extracted for enhancement. At that time the extracted image may be copied (or transferred) to a second memory device, and stored there for a relatively short time scale while an enhanced

version of the image is prepared. During the image enhancement procedure itself, calculation steps will be performed in which relevant data (for example Fourier data) is successively generated, stored in a third memory device, extracted from that memory device for use, and then deleted from that memory device. These operations may occur on a yet shorter time scale: for example in the time taken to process a small section of the image. The amount of data stored at any moment will vary, and may 10 not be determined by (or even related to) the size of the original image.

Furthermore, if the image enhancement algorithm is written in a high level programming language, there will 15 inevitably be many steps of data storage and data deletion performed by the operating system of the computer of which the user is unaware. Many of these data storage and deletion operations occur on a yet shorter time scale, and involve varying quantities of data.

20

With this example in mind, let us at once clarify the terminology used in this document.

The term "computational task" will be used here to 25 include any computing operation performed automatically or semi-automatically by a processor (which may be a single physical unit, or alternatively a distributed processor made up of a plurality of physically separate, or even spatially distant, units). For example, 30 "computational task" here includes image or sound generation or processing, processing of business or other financial data, text processing (for example, word processing or translation), etc.

The term "*data storage device*" includes any device that is capable of storing data, including devices which are internal or external to the unit including the processor. "Data storage device" thus includes a magnetic storage 5 device ( e.g. a hard-disk drive or floppy disk), an optical storage device ( e.g. a CD storage device, such as read/write CD disk system), an electronic memory device (such as an internal or external RAM memory), and even a *data storage device* which is a portion of an 10 integrated circuit which also carries some or all of the processor itself.

One or more *data storage devices* may together define a "data storage space". For example, the external RAM 15 memory of a conventional PC is a *data storage space*, normally supported by one or more integrated memory chips. The hard drive supports a second *data storage space*. And both of these *data storage spaces* are parts of a single *data storage space* which is the total *data 20 storage space* of the PC.

The term "*data item*" is used herein to include any data which is intended to be stored in the *data storage space*. A *data item* may have any length (for example use of the 25 plural word "data" does not imply that the *data item* consists of more than one bit) and any significance. For example, it may be a program (in any language), or any data to be subjected to computer processing, or which arises during computer processing.

30

The term "*data array*" is used to mean a section of a *data storage space* that is composed of one or more "fields" having a logical relationship to each other. A "field" may be of any size, corresponding to one or more data

items. No size limitation on a field is implied: for example, the size of a particular field may be one bit, one byte, one word, or longer. No limitation on the location of a field is implied: for example, a data array 5 may be composed of fields that are contiguously located in the data storage space, or may be composed of fields that are located at discontiguous locations on different data storage devices in the data storage space.

Similarly, no limitation on the location of the contents 10 of a field is implied: for example, a field may be composed of data items that are contiguously located in the data storage space, or may be composed of data items that are located at discontiguous locations on different data storage devices in the data storage space. The 15 logical relationship between the fields may be a linear one (for example each field may correspond to a respective section of a data storage space, and the order of the fields in the data array may correspond to the order of those sections within the data storage space ( 20 e.g. the order of the addresses), but any other logical relationship is also possible. For example, as discussed below in detail, the logical relationship between the fields may be hierarchical (e.g. the data array may be logically equivalent to a tree-like structure of fields).

25

Data items are either pre-existent within a data storage space or are stored by being written to locations within the data storage space. In a conventional computer system, selecting the locations for storage within the 30 data storage space is one of the tasks of a unit called an "allocator" (which may be one or more dedicated components, or implemented in software, e.g. as one of several functions performed by a CPU). The allocator selects locations in logical units of a "cell": a "cell"

is a data array .The computational task is performed by a unit called a "mutator" (which may be one or more dedicated components, or implemented in software, e.g. as one of several functions performed by a CPU). As the 5 mutator performs its computational task, data items are continually being written into the data storage space, and deleted. Thus at any moment the data storage space contains cells which are being used by the mutator for storing data (such an area is known as a "live cell"), 10 and cells-which are not being used by the mutator for storing data (such an area is known as a "free cell"). A live cell may also become a free cell without any deletion of the data items within that cell: this is either achieved through express communication between the 15 mutator and the allocator or through a process of usage analysis by a unit called the "garbage collector", or simply the "collector" (which may be one or more dedicated components, or implemented in software, e.g. as one of several functions performed by a CPU). A cell 20 which is not live but which contains data items that are still useful to the allocator or to the collector is called a "zombie cell", or simply a "zombie". In general terms, data storage using cells is referred to as *II heap storage*. The total size of a data storage space which is 25 available for heap storage will here be referred to as the quantity HEAPSIZE of that data storage space. The HEAPSIZE for a data storage space may vary during the course of the computational-task through the addition or deletion of sections of memory: such sections of memory 30 are called "chunks" and the heap storage so implemented is called a "chunked heap".

For most allocators, a cell includes zero or more fields for the data to be stored (i.e. a

data item) and zero or more fields for additional data which is used for reference purposes ( e.g. for the purpose of memory management itself). The latter form of data is here referred to as "reference data". For example, for a one-dimensional data storage space, each live cell may store reference data that includes the address in the data storage space of one of the fields of the cell. Further reference data may include a field indicating that the cell is in fact live, 10 a field indicating whether neighbouring cells in the data storage space are free or live, and a field indicating the locations of other live cells within the data storage space. For the avoidance of doubt, a free cell or a zombie cell may contain reference data even though it may 15 not contain a data item.

We define the " size" of a cell to mean the space within it which is useable for storing a data item. Thus, the " size" of a cell roughly 20 corresponds to the amount of the data storage space occupied by the cell, but the correspondence need not be exact due to the possibility of reference data. Size is an integer multiple of a fundamental data unit. Preferably, each cell will store reference data including 25 the size of said cell.

When a data item is to be stored, how should an allocator search for a suitable free cell to store it in? One simple strategy is for the allocator to search through 30 the data storage space (e.g. in address order) for the first free cell large enough to store the data item, and store it there. This is known as a " First Fit" method.

An alternative would be to search all available free cells and see if there is one cell of exactly the right size to store the *data item*. If there is not, the allocator can then search for the smallest cell that is 5 large enough to store the *data item*. This is called a "Best Fit" criterion. In the case that the *free cell* is larger than necessary to store the *data item*, the part of the *free cell* which is not used for storing the *data item* (if it is above a predetermined size) may become a new 10 *free cell*.

A further alternative is to search all available free cells to find the largest one, and store the *data item* there. Again, in the case that this *free cell* is larger 15 than necessary to store the *data item*, the part of the *free cell* which is not used for storing the *data item* becomes a new *free cell*. This is called a "Worst Fit" selection criterion. The rationale of the "Worst Fit" selection criterion is that the part of the *free cell* 20 which is not used for storing the *data item* will tend to be large, and thus "Worst Fit" will produce relatively large new *free cells*.

Which of the above criteria is optimal generally depends 25 upon the *computational task* in question (e.g. the distribution of the *data items*, both in time and size, and the order of allocation and freeing of *cells*). Preferably, a selection criterion should not lead to "fragmentation" (that is, the creation of a very large 30 number of small *free cells*). Also, preferably the time taken by the allocator should be "non-disruptive" (that is, it should never, even in a worst case, be unacceptably large) and should be "scalable" (that is, it should be independent of the value of *HEAPSIZE*).

A survey of the field of allocators is given in the paper "Dynamic storage allocation: a survey and critical review" by Paul R. Wilson, Mark, S. Johnstone, Michael 5 Neely, and David Boles (in the Proceedings of the International Workshop on Memory Management, vol 986 of Lecture Notes in Computer Science, Kinross, Scotland, September 1995, Springer- Verlag). As pointed out there, the efficiency of an allocator in storing memory in a 10 data storage space has major cost implications. Assuming, for example, that there are 100 million PCs in the world, each having 10 MBytes of memory at \$30 per Byte. If just one fifth is used for heap-allocated data, and one fifth of that is unnecessarily wasted, the cost is over a 15 billion dollars.

A further survey of the field was given by Thomas A. Standish in "Data Structure Techniques", Addison-Wesley, 1980. It is now generally accepted that *Best Fit* 20 allocation is one of the best strategies in practice (i.e. in most cases, on real loads, compared with other general approaches such as *First Fit*) for reducing the fragmentation that occurs in dynamic memory management of variable-sized cells. For example, Shore ("On the 25 external storage fragmentation produced by first-fit and best-fit allocation strategies", Communications of the ACM, 18(8): 433-440, August 1975) found that *Best Fit* and address-ordered *First Fit* allocation work equally well, and Wilson and Johnstone ("Real-time non-copying garbage 30 collection", in Eliot Moss, Paul R. Wilson and B. Zorn, editors, OOPSLNECOOP; '93 Workshop on Garbage collection in Object-Oriented Systems, October 1993) present evidence that *Best Fit* is "one of the best policies" for reducing fragmentation.

However, it is widely believed that *Best Fit* allocation is not scalable because the time taken to search for the *best-fitting free cell* must always be related to

5 HEAPSIZE. In the best case, it is assumed that the use of sorted trees can reduce the searching so that it scales with

the logarithm of HEAPSIZE.

10 A known allocator that uses a *Best Fit* criterion, called "Fast-fit" (described by Standish), will now be described in detail with reference to Fig. 1. This diagram shows a data structure for a memory with *free cells* of sizes 1, 1, 1, 1, 2, 2, 2, 4, 4, 5, 5, 5, 5, 8, 9, 9, 10, 27, 27, 77, 77 and

15 102. Each *free cell* contains at each end a pointer (shown in grey on the figure), pointing to another *free cell* of, the same size (where one exists) or to the other end of itself (if no other *free cell* of the same size exists).

Thus, the *free cells* contain enough information to define 20 "rings" of *free cells* of the same size. The allocator also uses a *data array* AVAIL having six fields. AVAIL(1)

is used to store the location of one of the four *free cells* of size 1, AVAIL(2) stores the location of a *free cell* of size 2. There are no *free cells* of size 3, and 25 this fact is indicated in AVAIL(3). AVAIL(4) and AVAIL(5) store the location of a *free cell* of size 4 and 5 respectively. Cells above size 5 are treated differently.

Specifically, for each ring, a representative cell is selected, and the representative cells are sorted into an 30 AVL tree.

To store a *data item* of size n, the following procedure is used.

- If n less than or equal to 5, and AVAIL(n) is not empty, the *data item* is stored in the *free cell*

indicated by AVAIL(n). If AVAIL(n) is empty, we search for the smallest j (larger than n) for which AVAIL(j) is not empty. If j is less than or equal to 5, the free cell indicated by AVAIL(j) is split into free cells of size n and (j-n), and the data item is stored in the cell of size n.

- 5 • If n is greater than or equal to 6 (or in the case in the last paragraph, if no j less than or equal to 5 can be found) we use n as a search key to search
  - 10 the AVL tree pointed to by AVAIL(6), and locate a ring of cells whose size s is the smallest that is also greater than or equal to n, and if s does not equal n then split it to form two free cells of size n and (s-n). The data item is stored in the free cell of size n thus found or created.
  - 15 • In all cases where an appropriate free cell is found, that cell is detached from its ring, with the pointers in the remaining cells being adjusted accordingly.
  - 20 • Whenever a cell is split, leaving a remaining free cell of size (j-n) or (s-n), we check to see whether the neighbouring cell to the remaining free cell is free or not. If it is, we detach the neighbour from its ring and coalesce the new free cell with that neighbour to form a larger free cell. The new free cell is inserted into the structure shown in Fig. 1.

25 The present inventor has investigated this scheme and has noticed a number of features of it. In general the time taken to identify a free cell for storing a data item rises (in the worst case) as the number of layers of the hierarchical tree. Since free cells may exist up

to any arbitrary size less than HEAPSIZE, and each ring in the tree may contain a single cell, and the tree has binary division, the number of layers in the worst case is of the order of  $\log_2(\text{HEAPSIZE}'')$ . Thus in the worst 5 case, the time taken for a memory storage task increases with increasing HEAPSIZE of the data storage space.

The present inventor has devised a variation of the Fast-fit algorithm in which it is ensured all free cells have 10 a size less than or equal to a maximum MAXFIELDS (the data

storage space may, for example, be generated with all cells less than or equal to this size, and coalescing of two cells may only be carried out in the case than a cell 15 larger than

MAX FIELDS is not created). In this case, the time taken, for a memory storage operation is in the worst case  $\log_2(\text{MAXFIELDS})$ , which is more acceptable since MAXFIELDS is not related to HEAPSIZE and may be very much smaller than 20 the square root of HEAPSIZE.

However, this estimate assumes that the AVL tree is approximately perfectly balanced. This would not be the case, for example, in Fig. 1 if the two cells of size 77 25 and one cell of size 102 were used up. As the tree becomes less balanced finding cells of a given size takes longer compared to the number of rings, and in the worst case, the time taken to find a free cell scales with the number of rings. To ensure that this does not occur, a 30 complex and time consuming occasional "re-balancing" algorithm would be required (re-balancing of an AVL tree is called "rotation"). While this operation is carried out, data storage is not possible, and thus the re-

balancing may cause unacceptable disruption of the computational task.

Any allocator gradually uses up the *free cells* in the  
5 data storage space, so, for memory storage to be sustainable, there must be a mechanism for generating new free cells. That is, when it is no longer required to store the data in a *live cell*, there should be a mechanism for converting the *live cell* into a *free cell*  
10 (i.e. for "freeing" cells).

In some systems, the freeing of memory is decided by the programmer. However, it  
is increasingly common for programming systems to offer  
15 automatic dynamic memory management including automatic freeing of cells that are no longer being used by the mutator; such cells are called "*garbage cells*". This is referred to here as "*garbage collection*", or simply "collection", and is performed by the collector (as  
20 previously defined).

In the implementation of functional programming languages, in particular, automatic detection and freeing of *garbage cells* is required. Preferably, collection should be  
25 scalable, and non-disruptive. Such collection overheads (which lead to short memory-management pauses due to allocation or collection) are vital for interactive and real-time applications.

30 Baker's incremental copying technique (Henry G. Baker, "List processing in real-time on a serial computer". Communications of the ACM, 21(4); 280-94, 1978) might be expected to form the natural basis for a collector with the required behaviour. However, the worst-case cost of

Baker's read barrier is potentially very high and very unpredictable.

Another sort of collection system (see G. E. Collins, "A  
5 method for overlapping and erasure of lists".

Communications of the ACM, 3(12):655-657, December 1960) is known as "reference counting" (i.e. counting the number of live cells which point to a particular cell - for each cell, this is called the "reference count" for 10 that cell). The reference count for each cell is normally held as part of the reference data for that cell. The advantages of reference counting are:

- it is naturally incremental (it is performed in small steps interleaved with computation);
- 15 • it is naturally distributed and can improve locality of reference.
- it is easy to implement;
- there is no root-set to be scanned (an important determinant of real-time performance);
- 20 • the overheads are predictably related to actual *mutator* activity rather than to the size of the heap or the numbers of live cells (many of which may be inactive for long periods);
- its behaviour does not degrade with increasing heap 25 occupancy;
- it is decoupled to a large degree from the *mutator*; synchronisation with the *mutator* is not required, and (unlike incremental tracing collectors) the collector is not fooled by *mutator* activity -the algorithm is 30 therefore simpler.

Consider the following three memory operations:

1. cell allocation ( and issuing of a pointer to the *mutator*)
2. pointer copy
3. pointer deletion

5

Using *reference counting*, the first two of these operations may be achieved with constant-time *garbage collection* overhead. However, a complication arises after 10 a *garbage cell* has been identified (always as the direct result of pointer deletion). If a cell's *reference count* drops to zero, it is garbage; before returning the *garbage cell* to the allocator's data structure for managing *free cells*, it is necessary to inspect each of this cell's *fields* and delete any pointers held therein. 15 We call this action '*freeing*' a field. The deletion of a pointer in this way might lead to another cell being identified as garbage, so potentially leading to a cascade of deletions. The time taken to service a deletion cascade is unpredictable and may be large. It is 20 therefore important that such cascades are avoided.

This problem is well-known and solutions have previously been proposed, for example by I. Wiesenbaum ("Symmetric list processor", Communications of the ACM, 6(9): 524- 25 544, September 1963).

- When a cell is identified as garbage it is placed on a "to be collected" data structure - these are *zombie cells*, as previously defined;
- The *fields* of *zombie cells* are freed incrementally, 30 interlaced with other *mutator* activity;
- When all *fields* in a *zombie* have been freed, the *cell* becomes free (it is passed to the allocator).

One option is to inspect *zombie fields* just prior to each cell allocation (since at this point we know that the

*mutator requires more memory).* If the freeing of a *zombie field* causes a pointer deletion, thereby causing another *cell's reference count* to become zero, then that *cell* simply becomes a *zombie* -the *fields* in the subsequent 5 *cell* are not inspected and no cascade occurs. This simple procedure eliminates cascades but introduces a further tension:

- If we guarantee always to free more *zombiefields* (if they exist) than the number of *fields* required for 10 the *cell allocation*, the *collector* always makes progress (*collection exceeds allocation* whilst there is *garbage* to be collected);
- If we guarantee always to free an exact number of *zombie fields* before each *allocation*, the *garbage collection overhead* for *cell allocation* is constant 15 (given a fixed *MAXFIELDS* limit).

In a system with variable-sized *cells*, it is not possible to guarantee both that the *collector* makes progress and that the *collection overhead per allocation* is a small 20 constant.

At the point of *cell allocation*, a *zombie cell*-should be freed. But which *zombie* should we choose?

- If we always free the largest *zombie* first, the resultant *free cell* can be used to satisfy many more 25 *allocation requests* than a small *cell*;
- By contrast, Last-In-First-Out (LIFO) freeing (i.e. the most recent) encourages depth- first freeing of cascades and might lead to better cache performance;
- However, it may be preferable to coalesce older 30 *cells* before younger *cells* using First- In-First-Out (*FIFO*) freeing, in order to improve fragmentation behaviour.

The choice of strategy affects the overheads for cell allocation. Both LIFO and FIFO zombie freeing can be achieved with constant time overhead per operation using a stack or queue, whereas largest-freeing requires that 5 the largest zombie be found. This is a search comparable in nature to the search described above in relation to free cells, and thus exhibits the same problems of scaling and possible disruption.

10 Summary of the invention

The present invention seeks to address one or more of the above problems, and to provide a novel and useful data structure.

15 Preferably, the present invention seeks to provide a data structure that makes possible a memory management system which is scalable, non-disruptive, and employs Best Fit allocation together with non-disruptive coalescing.

20 Preferably, the present invention makes it possible to achieve a (chunked) heap of variable-sized cells to be managed such that:

- Fragmentation is minimised, using Best Fit allocation (including cell-splitting and coalescing offree cells).
- 25 • The system is scalable, i.e. has memory-management overheads (in terms of memory accesses) being independent of the numbers of free cells or live cells, and independent of the size of the heap (which, with a chunked heap, may grow at runtime).
- 30 • Memory management is non-disruptive (for example, there are no "embarrassing pauses" whilst free cells are coalesced), and the three operations of cell freeing, cell allocation and cell coalescing each have a small bounded overhead in terms of memory accesses.

In a first aspect, the invention proposes in general terms that a data structure includes a first *data array* which indicates the existence of *cells* of a predetermined type (e.g. *free cells*) and of a predetermined size, and a second *data array* which indicates the location of such *cells* within the *data storage space*.

Specifically, in this first aspect the invention provides a data structure comprising:

- 10     • a *data storage space*;
- a first *data array* which, for each of one or more *cell sizes*, stores data indicating whether or not the *data storage space* includes at least one *cell* of a certain type and of the respective *size*; and
- 15     • a second *data array* which, for any said *size* for which at least one *cell* of the determined type exists, indicates a location of at least one *cell* of that *size* and *type* within the *data storage space*.
  
- 20     The first and second *data arrays* may each be sections (which may be allocated statically before the start of the computational task, or dynamically during the computational task, or comprise both statically-allocated and dynamically-allocated parts) of said *data storage space*, or sections of one or more additional *data storage spaces*. Though logically separate, the first and second *data arrays* may be merged to facilitate implementation.
- 25     The "type" of *cells* may be *free cells*, or alternatively *live cells*. However, the term "type" is not limited in this respect. For example, a "type" of *cells* may be *cells* which, irrespective of whether they are *live* or *free*, possess some additional characteristic, such as including certain data. Preferably, the *cell types* are predetermined, but alternatively it would be possible for

them to vary (e.g. be regularly reset) as part of an optimisation of the data structure in relation to the computational task in which it is employed.

One type of cell to which the invention is applicable, for example, is cells which do not contain data item(s) that are useful to the mutator but do contain pointers (to other cells) which may be useful to either the collector or the allocator. For example, zombie cells discussed above.

In the case that the type of cells is free cells (i.e. portions of the data storage space which are not presently being used by the mutator -for storing data), when it is desired to store a data unit of a certain size, the first data array can be used to identify a suitable cell size for storing it ( e.g. the smallest cell size which is at least as large as the data unit and for which at least one free cell exists), and the second data array can be used to locate a free cell of that size, so that the data unit can be inserted into it.

Preferably, said predetermined cell sizes are all no more than a predetermined maximum cell size (MAXFIELDS), which is independent of the total size of the data storage space (HEAPSIZE). For example, said predetermined cell sizes may be all cell sizes up to the predetermined maximum cell size. This means that the size of both the first and second data array may depend upon MAXFIELDS but be unrelated to HEAPSIZE.

Preferably, the first data array may be selected so that for a cell size n it is possible to determine the smallest (integer) value of p such that  $p > n$  and there exists at least one cell of the predetermined type of size p, within a time which scales with  $\log (MAXFIELDS)$ .

Alternatively, or additionally, the first data array may be such that for a cell size n it is possible to

determine the largest (integer) value of p such that  $p < n$  and there exists at least one cell of the predetermined type of size p, within a time which scales with  $\log(\text{MAXFIELDS})$ .

5 For example, the first data array may be a hierarchical data array, having:

- a first level which includes one or more first fields, each first field indicating for a respective size of cell the existence or otherwise of at least 10 one cell of that size, said first fields being divided into one or more groups, each group representing a respective range of cell sizes; and
- a second level which includes one or more second fields, each second field indicating for a 15 respective said group the existence or otherwise of at least one cell in the respective range of cell sizes.

We will refer here to the first layer as the "lowermost" layer, and to the second layer as a "higher" layer. We 20 will use the names "layer" and "level" synonymously. We will say that a particular cell size is "related" to any field of the first data array when that field either (i) contains information about the existence of cells of that size, or (ii) contains information about another field 25 which is itself related to the particular cell size. When for a given cell size there is at least one cell of the predetermined type having that cell size, we say that that cell size is "occupied". Similarly, when a field in the first data array indicates that one of the cell sizes 30 related to it is occupied, we will say that that field is "occupied".

Preferably, the first data array contains further layers "above" the second layer. In each i-th layer there are one or more fields. Unless the i-th layer is the top

layer, the one or more *fields* are divided into one or more groups, each group representing a range of *cell sizes*, and in the  $(i+1)$ -th layer there is a *field* for each respective group of the  $i$ -th layer. Thus, any 5 particular *cell size* is, in each layer, related to a single respective *field*. The existence of at least one *cell* of that particular *cell size* determines information in all *fields* related to it, in all layers of the *data array*.

10 If all groups contain two or more *fields*, at each level the number of *fields* is at most half the number of *fields* in the layer below. To be precise, if each *field* in an  $(i+1)$ -th layer represents a group of  $p$  *fields* in the  $i$ -th layer, the  $(i+1)$ -th layer contains  $p$  times as 15 many *fields* as the  $i$ -th layer. The largest *cell size* represented by the first *data array* is MAXFIELDS, so the, total number of layers scales as  $\log p$  (MAXFIELDS). This has the effect that a search within it for a *cell size* which is occupied, takes a time which scales as  $\log p$  20 (MAXFIELDS).

We will now give a single example of how this search may be carried out:

- 25 Suppose that for a given (unoccupied) *cell size*  $n$  we wish to find the lowest occupied *cell size* higher than  $n$ . To do this we begin at the lowest level of the table with the *field* related to  $n$ . We then ascend the table layer-by-layer.
- 30 • In each layer including the lowest we determine whether, in the group including the *field* related to *cell size*  $n$ , there is at least one *occupied field* representing *cells* larger than  $n$ .

- If so, then we select the smallest such occupied field, and move back down the layers, at each layer selecting the occupied field which is related to the selected field in the layer above and which represents the 5 smallest cells. The cell size we arrive at in the lowest layer-is the one sought.
- Otherwise, we go up one layer higher.
- If we reach the top layer, we check to see whether in that top layer there is at least one occupied field 10 representing cells larger than n.
- If so, then we select the smallest such occupied field, and move back down the layers, at each layer selecting the occupied field which is related to the selected field in the layer above and which represents the 15 smallest cells. The cell size we reach in the lowest layer is the one sought.
- Otherwise, no such cell exists.

Preferably, in each layer, each group represents a number 20 of fields in the layer below which is a multiple of eight (e.g. eight, sixteen or thirty-two). Occupancy or otherwise of a field may be represented by a binary digit. In this case, the group of fields may conveniently be represented as an integer number of bytes. In this 25 case, for many computer systems, the operations required for searching the tree can be carried out significantly more quickly (in real time), because the data array can be simply a bitmap.

If there are 32fields per group, then it is possible 30 to achieve the search in a time of  $O(\log_2 (\text{MAXFIELDS}))$ . If  $\text{MAXFIELDS}=32k$ ,  $\log_2 (\text{MAXFIELDS})$  is just 3, so the first data array has only three levels.

Although, as mentioned above, the predetermined cell sizes may be all cell sizes up to the value of MAXFIELDS, this is not in fact a necessary feature of the invention. A further option within the scope of the invention is to 5 treat cells in different ways according to their size. For example, it may be the case that due to fragmentation there will always in practice be a large number of very small cells, so that the first data array is not needed for such cells. Thus, it may prove more efficient for the 10 first data array only to be used for cells having a size which is at least a predetermined minimum size (here called "MINFIELDS"), and for cells smaller than MINFIELDS to be indexed differently. For example, the very small 15 cells might be treated in a manner similar to the Fastfit algorithm described above. That is, for cells smaller than MINFIELDS, the second data array could contain: (i), in the case of a cell size for which cells do exist, a pointer to such a cell, or (ii) in the case of a cell size for which cells do not exist, data indicating that 20 fact.

Thus, for example, in the case that the cells are free cells, and it is desired to locate a very small free cell (i.e. below the predetermined size) to store a small data unit, this could be done using the second data array 25 only. Provided that MINFIELDS is a value that is unrelated to either HEAPSIZE or MAXFIELDS (that is, it does not scale with HEAPSIZE or MAXFIELDS), the time scaling of the overall algorithm may be unaffected.

The existence of MAXFIELDS is not in fact a 30 necessary feature of the invention. A further option within the scope of the invention is to allow the first and second data arrays to contain fields for all cell sizes up to HEAPSIZE (which may vary). This option may be further optimised within the scope of the invention to

allow the first and second *data arrays* to contain fields for all *cell sizes* up to the largest *cell size* yet allocated at any given time during the performance of the computational task and for the size of the first and 5 second *data arrays* to be modified dynamically (including extension, contraction and re-balancing as necessary) during the computational task.

The invention has been explained above using a first *data array* which indicates the existence or otherwise of 10 a *cell* of an exactly predefined *size*. However, all of the above concepts are straightforwardly applicable to a case in which the *cell sizes* are in fact binned (i.e. conceptually divided into ranges), and the first *data array* is in fact used to indicate the existence of a *cell* 15 within a certain range.

Thus, the first aspect of the invention may alternatively be expressed as providing a data structure comprising:

- 20 • a *data storage space*;
- a first *data array* which, for each of one or more *cell size ranges*, stores data indicating whether or not the *data storage space* includes at least one *cell* of a certain type within the respective *size range*; and
- 25 • a second *data array* which, for any of said *size ranges* for which at least one *cell* of the determined type exists, indicates a location of at least one of those *cells* within the *data storage space*.

In the case that the *cell size ranges* are each only 30 one unit wide, this reduces to the expression of the first aspect of the invention given earlier. However, alternatively, any *cell size range* may be wider than one data unit (so that it includes two or more possible *cell*

sizes). The largest cell size range may be defined as all cells up to a predefined size (which corresponds to MAXFIELDS, or to HEAPSIZE in the option that MAXFIELDS is not used).

5 Preferably, the cell size ranges are predetermined, but alternatively it would be possible for them to vary (e.g. be regularly reset) as part of an optimisation of the data structure in relation to the computational task in which it is employed.

10 In either case, the width of a given cell size range may be a function of the lower limit of the cell size range. For example, the cell size range may for example be selected to be wider for higher cell sizes, since in that case the 15 inefficiency caused by the coarse graining of the cell sizes (i.e. the waste caused by replacing an "exact-fit", by an approximate fit) is only a small fraction of the size of the cell. This is in fact a further example of the option mentioned above, of treating cells of 20 different sizes differently.

Preferably, the cell types are predetermined, but alternatively it would be possible for them to vary (e.g. be regularly reset) as part of an optimisation of the data structure in relation to the computational task in 25 which it is employed.

As mentioned above, all preferable features of the data structure according to the first aspect of the invention are applicable within the scope of the invention to the case in which the cell sizes (whether 30 predetermined or varying)-are generalised to cell size ranges. However, for simplicity of terminology, the following explanation is given on the basis that the cell sizes are predetermined as exact values (i.e. not ranges). Similarly, for simplicity of terminology, the

following explanation is given on the basis that the cell types are predetermined.

The data structure preferably includes a field which stores the location of at least one of: (i) a cell of the predetermined type having the greatest size (among the predetermined sizes) for which cells of the predetermined type exist (this field is herein called "MAXP"), or (ii) a cell of the predetermined type having the smallest size (among the predetermined sizes) for which cells of the predetermined type exist (this field is herein called "MINP").

MAXP is useful, for example, in the case that the predetermined cell type is a free cell, because it means that the *Worst Fit* method can be used for selection of a free cell to hold a data item. MAXP is also useful in the case that the predetermined cell type is a *zombie cell*, because in that case it is easy to find the largest zombie cell to free.

Furthermore, in the case that the predetermined type of cell is a free cell, if it is determined (e.g. using the first data array) that no cell sizes equal to or less than n are occupied, then MINP points to a free cell which (among the extant free cells) is of the smallest size capable of storing a data item of size n.

Preferably, the cells of the predetermined type contain pointers to (i.e. data indicating the location of) each other. A pointer is a further example of reference data. For example, each cell may contain at least two such pointers.

Preferably, pointers of the cells of each predetermined size are such that, starting from any cell of any predetermined cell size, any other cell of the predetermined type and the same cell size can be

reached by moving successively from one cell to another according to the pointers.

Preferably, at least one cell of each predetermined size contains a pointer to one of the cells of the next 5 higher occupied predetermined cell size. Preferably, at least one cell of each predetermined size contains a pointer to one of the cells of the next lower occupied predetermined size. If both of these conditions are met, then it is possible to move between all cells of the 10 predetermined type having any of the predetermined sizes using pointer information alone.

For example, in the case that the predetermined type of cell is a *zombie cell*, it is possible to start at the cell indicated by MAXP, and move through all the *zombie* 15 cells in an order in which the size of the *zombie* cells never increases (i.e.. only decreases or remains the same), using the pointers. This possibility may be particularly useful for *garbage collection*.

The order of the path defined by the pointers 20 through cells of the same size may encode information about the cells of that size, for example the age of the cells (i.e. the time since they were created).

For example, in the case of a path towards lower cell sizes, the cell of any given size which is indicated 25 (pointed at) by a cell of the next higher size, may be the oldest cell of the given size, and the cells along the path towards the cell which points to the next lower size may be progressively younger. This age order makes it easy to locate old (or young) cells of a given type 30 and size (for example, for conversion to another type of cell). The age order may be preserved by, whenever a new cell of the given size is created, changing the pointer in the youngest cell of the given size to point at the

new cell, and arranging for the new cell to contain a pointer to a cell of the next lower size.

Clearly, the scheme of the preceding paragraph may be varied within the scope of the invention, to provide a 5 data structure in which the path leads from young cells to old cells, and/or to the case that the path leads from small to large cells.

Preferably, whenever a first cell of the predetermined type contains a pointer to a second cell of 10 the predetermined type (of the same or different size), that second cell contains a pointer to the first cell. This has the effect that the path may be followed in either direction (i.e. towards either larger or smaller cells).

15 Although in principle the second data array may store the location of any (or all) of the cells of the predetermined size (or size range), it is preferable that the (at least one) cell of each size (or size range) is selected according to a characteristic of the cell. For 20 example, a further option within the scope of the invention is for the second data array to contain, for each cell size or cell size range, the locations of both the youngest and the oldest cells of the predetermined type and of the predetermined size. Alternatively, or 25 additionally, the second data array may contain, for each cell size range, the locations of both the biggest and the smallest cells of the predetermined type and of the predetermined size.

A straightforward way of realising MINP is for the 30 data structure to include at least one cell of the predetermined type (herein referred to as a "sentinel cell") which is of size MINSIZE or less (e.g. it might be of size zero) having a predetermined location in the data storage space and containing a pointer to a cell of

the predetermined type of the next higher size. In other words, a pointer to the sentinel cell thus constitutes the MINP field. Since the sentinel cell exists, a search using the first data array for a cell of the 5 predetermined type of highest size lower than size n will find the sentinel cell if no cell of intermediate size exists, and the pointer in the sentinel cell leads to a cell of the minimum size greater than n.

In principle, a sentinel cell with size MAXFIELDS 10 and a predetermined location could additionally, or alternatively, be provided in the data storage structure.

A further option within the scope of the invention is for either the MAXP field or the 15 MINP field or both fields to be contained in the second data array.

In fact, the concept of using pointers as described above constitutes an independent second aspect of the invention.

20 Specifically, in the second aspect the invention proposes a data structure including:

- a data storage space, the data storage space comprising one or more cells of a certain type, each said cell of said type containing at least one pointer to the 25 location in the data storage space of another said cell of said type; and
- a field which stores as one or more pointers the location within the data storage space of at least one of: (i) one of said cells of said type which is at 30 least as large as any other of said cells and of said type, or (ii) one of said cells of said type which is at least as small as any other of said cells and of said type;

- said pointers being such that, starting from a cell of which the location is indicated in said field, any other of said cells can be reached by moving successively from one cell to another according to the 5 pointers in a path in which the size of the cells varies monotonically.

By "monotonically" is meant that the size of the cells in the path either (i) never falls 10 ( e.g. in the case of starting with MINP), or (ii) never rises ( e.g. in the case of starting with MAXP).

For example, at least one cell of each size may contain a pointer to a first one of the cells of the next higher (or lower) second cell size, and the pointers 15 within the cells of the second size may define a path from the first cell through all the other cells of the second size to the cell of the second size which contains a pointer to a cell of the next higher (or lower) cell size.

20 Preferably, as in the first aspect of the current invention, the cell types are predetermined, but alternatively it would be possible for them to vary (e.g. be regularly reset) as part of an optimisation of the data structure in relation to the computational task in 25 which it is employed.

The invention is defined above in terms of data structures. However, in third and fourth aspects, the invention in general terms respectively proposes using a 30 data structure according to the first aspect or second aspect of the invention in a memory management method. The data structure may have any of the preferable features described in detail above in relation to the first aspect or second aspect of the invention.

Specifically, in the third aspect the invention provides a memory management method for controlling one or more *data storage devices* which support a *data storage space*, the memory management method including:

- 5     • deriving a first *data array* which, for each of one or more *cell sizes*, stores data indicating whether or not the *data storage space* includes at least one *free cell* of the respective *size*;
- 10    • deriving a second *data array* which, for any said *size* for which at least one *free cell* exists, indicates a location of at least one *free cell* of that *size* within the *data storage space*; and
- 15    • upon receiving an instruction to store a *data item*, determining using said first *data array* an appropriate *cell size* of a *free cell* for storing the *data item*, and using said second *data array* to derive the location within the *data storage space* of a location of a *free cell* of the determined *cell size*.

20    Preferably, the *data item* will be stored in the said *free cell*, the said *cell* will be converted into a *live cell*, and the first and second *data arrays* will be updated to indicate that said *cell* is no longer *free*.

25    Preferably, upon receiving an instruction to add a new *free cell* of a certain *size*, the location of said *cell* will be added to the appropriate *field* in the second *data array*, said *cell* will be converted to a *free cell*, and the related *fields* of the first *data array* will be updated to indicate that 30 a *free cell* of said *size* exists.

No limitation is implied on the ordering of the operations in this method: operations may be ordered

sequentially or non-sequentially except where constrained by logical dependencies. :

In a fourth aspect the invention provides a memory management method for controlling one or more data storage devices which support a data storage space, the data storage space containing:

- one or more free cells, each said cell containing at least one pointer to the location in the data storage space of another said cell; and
- 10 • a field which stores the location within the data space of at least one of: (i) one of said cells which is at least as large as any other of said cells, or (ii) one of said cells which is at least as small as any other of said cells;
- 15 • said pointers being such that, starting from a cell of which the location is indicated in said field, any other of said cells can be reached by moving successively from one cell to another according to the pointers in a path in which the size of the cells varies monotonically;
- 20 • the method including identifying a free cell of a certain size by starting from a first free cell, and moving successively from one cell to another according to the pointers until a cell of that size is reached.

Preferably, upon receiving an instruction to store a data item of a certain size, an appropriate free cell will be determined using the method described in the fourth aspect and starting the search from the said 30 field, the data item will be stored in the said free cell, the said free cell will be converted into a live cell, and the cell will be detached from the linked-list of free cells.

Preferably, upon receiving an instruction to add a new *free cell* of a certain *size*, said *cell* will be converted to a *free cell* and added to the linked-list of *free cells* in the appropriate position such that the properties expressed in the fourth aspect are maintained (i.e. the pointers provide a path in which the size of the *cells* varies monotonically).

No limitation is implied on the ordering of the operations in this method: operations may be ordered sequentially or non-sequentially except where constrained by logical dependencies.

Preferably, the first and fourth aspects of the invention will be combined so that, upon receiving an instruction to add a new *free cell* of a certain *size*, or to allocate a *data item* of a certain *size*, the appropriate position for adding the new *free cell* to or deleting an existing *free cell* from the linked-list of *free cells* may be determined by inspecting the first and second *data arrays* and said *data arrays* will be updated accordingly to indicate either the presence of a new *free cell* or that an existing *free cell* has been deleted. This has the effect that both adding a new *free cell* and allocating an existing *free cell* can be achieved in O ( $\log p(\text{MAXFIELDS})$ ) time.

In both the third and the fourth aspects of the invention, the memory management method preferably further includes a step of identifying at least one *cell* of a type other than a *free cell* (e.g. a *live cell* or a *zombie cell*) and converting it into a *free cell*.

Preferably, when a said *cell* is converted into a *free cell*, and if the *reference data* required for a *free cell* exceeds the available fields for *reference data* in the said *cell*, then the *fields* of the said *cell* that were used for *non-reference data* may be used to store the

reference data for the free cell after conversion. Preferably also, when a free cell is converted to a live cell as the result of allocation, those fields in the free cell that were used for reference data may be used 5 for non- reference data.

For this purpose the memory management method preferably employs a data structure according to the first or second aspect of the invention in relation to cells of this other type.

10 In a fifth aspect, the invention proposes a memory management method for controlling one more data storage devices which support a data storage space containing one or more zombie cells, each zombie cell containing at least one pointer to the location in the data storage 15 space of another zombie cell;

- the method employing a field which stores the location within the data storage space of one of said zombie cells which is at least as large (or as small) as any other of said zombie cells;
- 20 • said pointers being such that, starting from a zombie cell of which the location is indicated in said field, any other of said zombie cells can be reached by moving successively from one zombie cell to another according to the pointers in a path in which the size 25 of the zombie cells varies monotonically;
- the method including freeing zombie cells in an order defined by starting from a first zombie cell, and moving successively from one zombie cell to another according to the pointers.

30 Preferably, upon receiving an instruction to free a zombie cell, an appropriate zombie cell will be determined using the method described in the fifth aspect and starting the search from the said field, the said

*zombie cell will be converted into a free cell, and said free cell will be detached from the linked-list of zombie cells.*

Preferably, upon receiving an instruction to add a new *zombie cell* of a certain *size*, said cell will be converted to a *zombie cell* and added to the linked-list of *zombie cells* in the appropriate position such that the properties expressed in the fifth aspect are maintained (i.e. the pointers provide a path in which the *size* of the *cells* varies monotonically).

No limitation is implied on the ordering of the operations in this method: operations may be ordered sequentially or non-sequentially except where constrained by logical dependencies.

Preferably, the first and fifth aspects of the invention will be combined so that, upon receiving an instruction to add a new *zombie cell* of a certain *size*, or upon receiving an instruction to free a *zombie cell*, in the former case the appropriate position for adding the new *zombie cell* to the linked-list of *zombie cells* may be determined by inspecting the first and second *data arrays* and in both cases said *data arrays* will be updated accordingly to indicate respectively either the presence of a new *zombie cell* or the deletion of a *zombie cell*. This has the effect that adding a new *zombie cell* or freeing the largest *zombie cell* can be achieved in  $O(\log p(\text{MAXFIELD}))$  time.

In a sixth aspect, the invention proposes that the concepts described above are generalised by replacing cell size with any integer-valued property associated with a cell.

Specifically, in this a sixth aspect, the invention proposes a memory management method for controlling one

more data storage devices which support a data storage space containing one or more cells, each cell containing at least one pointer to the location in the data storage space of another cell, and each cell additionally associated with (e.g. containing) at least one integer quantity or property (which we call the "sort value");

- the method employing a field which stores the location within the data storage space of one of said cells which contains a sort value that is at least as large (or at least as small) as any other sort value of said cells;
- said pointers being such that, starting from a cell of which the location is indicated in said field, any other of said cells can be reached by moving successively from one cell to another according to the pointers in a path in which the sort value of the cells varies monotonically;
- the method including deleting cells in an order defined by starting from a first cell and moving successively from one cell to another according to the pointers.

Preferably, upon receiving an instruction to delete a cell, an appropriate cell will be determined using the method described in the sixth aspect and starting the search from the said field, and the said cell will be detached from the linked-list of cells.

Preferably, upon receiving an instruction to add a new cell of a certain sort value, said cell will be added to the linked-list of cells in the appropriate position such that the properties expressed in the sixth aspect are maintained (i.e. the pointers provide a path in which the sort value of the cells varies monotonically).

No limitation is implied on the ordering of the operations in this method: operations may be ordered

sequentially or non-sequentially except where constrained by logical dependencies.

Preferably, the first and sixth aspects of the invention will be combined so that, upon receiving an instruction to add a new *cell* of a certain *sort value*, or upon receiving an instruction to delete a *cell*, in the former case the appropriate position for adding the new *cell* to the linked-list of *cells* may be determined by inspecting the first and second *data arrays* and in both cases said *data arrays* will be updated accordingly to indicate respectively either the presence of a new *cell* or the deletion of a *cell*. This has the effect that adding a new *cell* or deleting the *cell* with the largest *sort value* can be achieved in  $O(\log p(MA.XFIELDS))$  time.

The above defined sixth aspect of the current invention thus provides a "priority queuing" mechanism for sorting *data items* of a certain type with respect to any integer quantity or property of said *data items* or of the *cells* that contain said *data items*. A "priority queue" is a data structure and associated methods which support the insertion of *data items* in any order and the selection (i.e. deletion) of *data items* in a predetermined order (for example, in order of the size of the *cells* that contain said *data items*, or in order of the value of the *data item*, which may or may not be numerical). Thus, it is a mechanism for sorting *data items* into a certain order. Priority queues are important for a wide range of computational tasks: for example, the scheduling of processes by an operating system, the routing of prioritised data in a communications network (for example, the Internet), the servicing of prioritised requests by an Object Request Broker, and the processing of data from multiple sensors (such as found, for example, in aircraft, spacecraft and self-navigating

missiles). When said computational task has real-time constraints, the time performance of the priority queue is a critical element of the successful implementation of that task.

5 Priority queues typically exhibit insertion and deletion time overheads that either both scale logarithmically with the number of cells in the queue, or one scales linearly with the number of cells in the queue and the other takes constant time. By contrast, the sixth  
10 aspect of the current invention provides a priority queuing mechanism that has deletion and insertion time overheads that scale as follows:

- where MAXFIELDS and MINFIELDS limits are set,  
 $O(\log_2 (\text{MAXFIELDS} - \text{MINFIELDS}))$ ;
- 15 • where no limit is set, and the data arrays are not dynamically re-balanced,  $O(\log_2(\text{maximum cell value}))$ ;
- where no limits are set, and the data arrays are dynamically re-balanced,  $O(\log_2(\text{number of different cell values}))$ .

20 This has the effect that insertion and selection may be much faster than conventional priority queues in situations where there are many duplicates and where the values are clustered.

25 A further useful property of the current invention is that it allows data items of equal value (in terms of the criterion for selection) to be selected in order of age (either oldest-first or youngest-first). Most implementations of priority queues do not provide this  
30 facility. This has the effect that stronger guarantees can be provided about the behaviour of computer software that uses the current invention as the basis for priority queuing.

In further aspects the invention provides memory management methods (fully combinable with the methods according to any of the third to sixth aspects of the invention) in which a data structure according to the 5 first or second aspect of the invention is updated by the insertion into it of a cell of the certain type, and the first and second tables and any pointers are updated accordingly. Preferably, this insertion preserves any information about the cells encoded by the pointers.

10

In further aspects, the invention provides a *data storage device* which is arranged to store a data structure according to the first and/or second aspects of the invention, and/or to perform any of the methods 15 according to the invention (including means for performing each respective step of the methods). The invention further provides a computer system for performing a *computational task* and including such a *data storage device*.

20 Any of the above aspects may be fully combined, as demonstrated in two examples of the invention that will now be described in detail for the purpose of example only.

25 Detailed Description of two examples of the invention

The first example describes a priority queue that does not limit the sizes of the first and second data arrays (i.e. there is no MAXFIELDS limit). The second example 30 describes a memory allocation system where the sizes of the first and second arrays are limited by MAXFIELDS.

Priority Queue

In this example the first and second data arrays of the first aspect of the current invention are merged for ease of implementation. The result is a single data array that contains both those fields determining the existence 5 of cells of a certain value and those fields determining the location of said cells if they exist.

The data array comprises a hierarchy of memory "blocks". Each block consists of a bitmap (in this example, an unsigned 32-bit integer) and one or more 10 fields held in contiguous locations (so that standard array indexing may be used to locate afield in constant time), with each field containing a pointer (in this example each block contains 32 pointers). At the lowest level, the pointers give the locations of cells: at all 15 other levels the pointers hold the addresses of blocks in the next lower level. Additionally the data array contains afield holding a pointer to the top layer of the hierarchy, and afield that stores the current number of layers in the hierarchy. The bitmaps provide the 20 implementation of the first data array of the first aspect of the current invention and the pointers provide the implementation of the second data array of the first aspect of the current invention.

Initially there exists a sentinel cell that had a 25 known fixed address and is never subsequently moved or deleted. Each cell contains reference data including (i) the value of said cell to be used for ordering subsequent selection, and (ii) a pointer to the "next" cell of lower or equal value (if such a cell does not exist, this field 30 is empty), and (iii) a pointer to the "previous" cell of higher or equal value (if such a cell does not exist, the pointer gives the location of the sentinel cell). Thus, the cells are arranged as a double-linked list. The double-linked list is at all times maintained in

monotonic order of cell value. This arrangement provides the implementation of the second aspect of the current invention.

To provide an implementation of a priority queue, 5 the first and second aspects of the current invention are combined as follows. The pointers of the lowest level of the data array point to cells in this list -there is a single pointer from the data array for each different value that is currently being used for sequencing in the 10 priority queue. For a certain value, if duplicate cells with that value exist, they are grouped next to each other in the double-linked list and the pointer from the data array for that value gives the location of the cell which is linked to the 15 cell with the next highest value (or to the sentinel cell, if no higher valued cell exists). This is illustrated in Figure 2.

The priority queue is initialised as follows:

- The sentinel cell is empty and there are no other 20 cells;
- The data array has one block containing 32 fields and a bitmap, each field being NULL and each bit in the bitmap being 0;
- The pointer to the top layer of the data array (we 25 call this variable "top") contains the location of the first field (that which points to the sentinel cell);
- The variable which gives the number of layers in the data array (we call this variable "layers") is set 30 to the value 1;
- Upon receipt of an instruction to insert a new cell into the priority queue, the following actions occur:

- The reference data of said cell is inspected to determine the value that will be used for subsequent selection -we call this value "x", We call said cell "this cell" and its previous and next pointers are called "previous" and "next",  
5

- First we determine whether the data array has sufficient layers to support the insertion, If not, we create new layers:

```
while (x >= (32^layers)) {  
  
10    ptr = new (block) ; /*initialised empty */  
        ptr[0] = top;  
        top = ptr;  
        layers = layers + 1;  
    }  
  
15    • Next, the following code performs an O(log N)  
        search of the data array, where N is the largest  
        value of any cell in the priority queue. If blocks  
        are missing from the hierarchy, they are created.  
        a = top;  
  
20    b = layers;  
  
while (b > 1)  
    c = x div (32^(b-1)) -(32^(b-1)) *(x div  
    (32^b));  
    if (a[c] == NULL) {  
  
25    ptr = new (block) ; /*initialised with  
        NULLs */  
        a[c] = ptr;  
    }  
  
30    a = a [c] ;  
    b = b -1;  
}  
  
result = a[x mod 32] ;
```

- If the result is not empty (the pointer is not NULL) then simply de-reference the pointer to find the cell and link the new cell into the double-linked list of cells (the previous "result" pointer should be updated to point to the new cell). Otherwise update the "result" pointer to point to the new cell and undertake a search of the bitmaps in the data array to determine the previous and next cells in the double linked-list to which the new cell should be linked, and to set the appropriate bits in the related bitmaps to indicate the existence of the new cell (this procedure is explained in the second example, below, which considers a memory allocation system).

Upon receipt of an instruction to select (delete) the largest valued cell, this is achieved in  $O(\log N)$  time (as with insertion) by (i) following the sentinel cell pointer to find the largest cell, (ii) detaching the largest cell by adjusting the sentinel pointer and the "previous" pointer in the next cell as necessary, (iii) traversing the data array from the top layer to the lowest layer to update the appropriate bits in the related bitmaps, and (iv) having reached the lowest level, updating the pointer in the data array (either to set it to NULL or to set it to point to the next cell of the same value as the cell just deleted).

### 30 Memory Management Method

This example of the invention is a memory allocation system where the sizes of the first and second arrays are limited by MAXFIELDS.

The memory allocation system provides best-fit memory allocation with allocation delays that are independent of the size of the memory.

5

The memory is divided into blocks. Each block comprises both an administrative data area and a user data area. If the block is not being used by the program (known as a "free" block), the user data held in the user data area 10 is irrelevant and the user data area may therefore be used to hold additional administrative data. If the block is being used by the program (known as a "live" block) then the user data area may not be used for administrative data.

15

The administrative data for all blocks is the same and comprises:

- (i) the size (the number of bytes) of the user data area
- 20 (ii) whether the block is free or live
- (iii) whether the previous block (the block occupying an adjoining position in memory at a lower memory address) is free or live
- (iv) the size (the number of bytes) of the user data 25 area of the previous block

The sizes are expressed as numbers of bytes. The size of the user data area for a live block may not exceed MAXFIELDS bytes. The size of a free block may, however, 30 exceed MAXFIELD bytes (as described below).

When the system starts, the memory contains amongst other things a single very large free block that represents the entire memory that the program may use. The size of this

block (to which we give the name "wilderness block") will be bigger than MAXFIELDS. Also, during the course of the program run, adjacent free blocks may be coalesced, and may thereby create a free block whose size is bigger than 5 MAXFIELDS (we also give these the name "wilderness blocks").

This example describes a memory management system where MAXFIELDS is less than 32,768 and therefore both the availability (free or live) and the size of a block can 10 be expressed with a 16 bit signed integer. The sign bit indicates whether the block is free (negative) or live (positive) and the magnitude indicates the size. An integer value of -32,768 indicates a free block that is bigger than 32,767 bytes and the actual size of the free 15 block is stored in the data area of that block (see below).

Each free block contains, in addition to the required administrative data of two 16-bit integers (one for size 20 and availability of this block, and one for the size and availability of the previous block), two pointers to other free blocks. The first such pointer (to which we give the name "leftp") gives the memory address of the free block that is next highest in a double-linked chain 25 of free blocks. The second such pointer (to which we give the name "rightp") gives the memory address of the free block that is next lowest in the double-linked chain of free blocks. These two pointers are held at the start of the user data area of a free block. Each pointer 30 requires 4 bytes and so the user data area of a block in this system must always be at least 8 bytes. If the program requests a block of less than 8 bytes user data area, it will be provided with a block of 8 bytes user data area (a request for a negative size is treated as an

error). This introduces a small amount of internal fragmentation. Wilderness blocks hold the actual size of the user data area in the user data area itself, immediately after the "rightp" pointer. A copy of the 5 size is also held at the very end of the user data area (to assist coalescing of free blocks).

In summary, the three types of block have the following layout:

10

1. live block

administrative data area:

2 bytes - size and availability of this block -

15 must be positive

2 bytes - size and availability of previous block

user data area:

N bytes - user data area, where N >= 8

20 2. free block (not wilderness)

administrative data area:

2 bytes - size and availability of this block -

must be negative

25 2 bytes - size and availability of previous block

user data area:

4 bytes - "leftp" - memory address of next block

in free list

4 bytes - "rightp" - memory address of previous

30 block in free list

N bytes - user data area, where N >= 0

3. wilderness block (free)

administrative data area:

2 bytes - size and availability of this block -  
must be -32,768

2 bytes - size and availability of previous block

5 user data area:

4 bytes - "leftp" - memory address of next block  
in free list

4 bytes - "rightp" - memory address of previous  
block in free list

10 4 bytes - actual size of user data area

N bytes - remainder of user data area - where N >=  
32,752

4 bytes - actual size of user data area

15 At initialisation, the memory comprises three blocks:

- a "sentinel" block (a live block used by the system) at  
the lowest address
- a "max" block (a live block used by the system)
- a "wilderness" block (available to the program)

20

The system also maintains:

(i) an array (to which we give the name "pgarray") of  
MAXFIELDS+1 pointers to free blocks that are not  
25 wilderness blocks. For administrative purposes the first  
pointer in this array (corresponding to a free block of  
size zero, which cannot exist) is initialised to contain  
the memory address of the "sentinel" block.

(ii) a pointer to the "max" block (which may, but need  
30 not, be held in the array described in (i), since entries  
1-7 of that array will be unused)  
(iii) a 3-level hierarchy of bitmaps containing  
information about the data held in the array described in  
(i). There need only be three levels if MAXFIELDS is

- less than 32,768. The first level of bitmaps is an array (to which we give the name bitmap0) of 1024 unsigned integers. This will support up to 32768 entries in pgarray. The second level of bitmaps is an array (to 5 which we give the name bitmap1) of 32 unsigned integers; this will support up to 1024 entries in bitmap0. The final level of bitmaps is a single unsigned integer to which we give the name bitmap2; this supports up to 32 entries in bitmap1.
- 10 (iv) a linked-list of free blocks in size order, with the lowest block in this linked list being the "sentinel" block and the highest block in this linked list being the "max" block. Note that the "sentinel" and "max" blocks are both live blocks, yet they are the two ends of the 15 double-linked free-list. It is important that they are live, not free (unlike all other blocks on the free list) because they must never be allocated and they must never be coalesced with other blocks.
- 20 If this memory management system is to be used for a program that allocates only individual bytes of data, then fragmentation may be minimised by permitting the system to allocate blocks starting at any address in memory. However, if the program allocates larger units of 25 data such as integers then additional internal fragmentation will occur because the system will be required only to allocate blocks where the user data starts at a memory address that is a multiple of 4 (alternatively, 8-byte or 16-byte alignment may be 30 required); to achieve this, it is necessary to ensure that (i) the initial wilderness block has a user data area that starts at a suitably aligned memory address and (ii) all subsequent requests for block allocation are rounded up to the nearest size such that the neighbouring

free block (at higher address) has a user data area that is correctly aligned.

The method proceeds as follows:

5

1. an initialisation method ensures that:

- the bitmaps firstly have all bits set to zero
- the bitmaps then have the first bit at each of the three levels set to 1 (this is because pgarray will 10 be initialised with pgarray[0] containing the address of the sentinel block)
- pgarray[0] is given the memory address of the sentinel block, which is defined to be at the lowest end of memory
- 15 • the sentinel block followed by the max block are allocated at the lowest end of memory alternatively, the max block could be placed at the upper end of memory). The "previous size and availability" data held in the max block are set to the appropriate data 20 for the sentinel block.
- two variables with the names "sentinelp" and "maxp" are set to contain the addresses of the sentinel block and the max block
- a wilderness block is created in memory immediately 25 after the max block, occupying all of the rest of the available memory. Because it is a wilderness block, pgarray does not contain an entry for this block. However, the block must be linked into the double-linked free list. The "previous size and
- 30 availability" data for the wilderness block are set to the appropriate values for the max block. The "size and availability" data for the wilderness block are set to -32,768.

- the leftp for the sentinel block and the rightp for the max block are both set to the memory address of the wilderness block
  - the rightp of the sentinel block and the leftp of the max block are both set to zero
  - the leftp of the wilderness block is set to the address of the max block, and the rightp of the wilderness block is set to the address of the sentinel block.
  - the size of the wilderness block is stored in the wilderness bloc just after the rightp and a copy is stored at the end of the user data area of the wilderness block
2. A main method receives memory requests from the program and allocates memory appropriately. Three different requests may be received: "malloc", "realloc" and "free".

The first is a request for the allocation of new memory of a given size; the main method finds the free block that is closest to the given size and splits that block into two parts - the lower part has a user data area that is the size required by the program (subject to alignment constraints as indicated above) and its memory address is returned to the program, whilst the upper part is retained as a free block for future use. The third request is that a live block at a given address should no longer be considered as live - it is therefore linked into the free list so that it may be available for future use. The second request is that a live block should be resized (either to a greater or smaller size), with the data in the current block being retained - this may be achieved by growing or shrinking the existing live block, or by allocating a new block, copying the data, and

freeing the existing block. For each request, the main method passes control to a subsidiary method to deal with the request. The three subsidiary methods are given the names "ccmalloc", "ccrealloc" and "ccfree". These three 5 methods are described in detail below.

### 3. The method "ccmalloc"

This method is provided with the size of user data area 10 that is requested. The method allocates the best-fit free block, unlinks that free block from the double-linked chain of free blocks, resets the pgarray and bitmaps as necessary, updates the administrative data for the block to show that it is now a live block, and returns the 15 memory address of the block. If the selected free block is larger than the requested size, the remainder of the block may be divided from the requested part and the remainder is then treated as a new free block and linked into the double-linked chain of free blocks and the 20 pgarray and bitmaps are updated as appropriate.

When ccmalloc is called for the first time, there will be no entries in pgarray apart from the pgarray[0] entry 25 that points to the sentinel block. The leftp pointer of the sentinel block points to the wilderness block, which can then be split to satisfy the request.

The method is described in detail as follows:

- 30 • first the requested size is set to 8 if it is less than 8
- if the size is greater than MAXFIELDS, the allocator terminates with an error

- if pgarray[size] is not empty then a free block of the requested size is available; this block is changed from being a free block to a live block by passing the memory address of the identified block as an argument (together with the requested size) to a method with the name "ccdelete" (this method is described below).  
5 The address of the block is then returned as the result of this method "ccmalloc"
- if pgarray[size] is empty then no free block of the requested size exists, and so a search must be made for the next biggest free block.
- control is passed to the method "search"; with the requested size passed as an argument. The "search" method is described below. It returns the index in the pgarray corresponding to the next smallest free block
- the leftp of the next smallest free block contains the memory address of the next biggest free block. This is the block that will be used to satisfy the request. We name this the "target" block
- if the target block is a wilderness block then we chose a new target block which is the wilderness block whose address is held in the rightp of the max block. This switch (together with appropriate actions when free blocks are coalesced) ensures that those wilderness blocks that are created by coalescing are chosen to fulfil allocation requests in preference to the wilderness block at the end of memory - this in turn reduces the overall memory requirements of the program.  
20  
• the amount of memory to be taken from the wilderness block will be the requested size (the administrative data at the start of the wilderness block can be reused for the newly-allocated block)

- the size of the remaining block will be the current size of the target block less the requested size less the size of the administrative data for the remaining block - we give this value the name "j"
- the administrative data for the remaining block is set such that the "previous availability and size" is positive and is the requested size
- the administrative data for the newly allocated block is set such that "my size an availability" is positive and is the requested size
- if the size of the remaining block is less than 32,768 then the administrative data for the remaining block is set such that "my size and availability" is negative and has the value given by "j"
- and the memory address of the remaining block is passed as an argument to the method "ccfree" if the size of the remaining block is greater than 32,767 then the administrative data for the remaining block is set such that "my size and availability" is the value -32,768 and the real size given by "j" is stored just after the rightp for the remaining block and a copy of "j" is stored at the end of the user data area of the remaining block; the leftp for the remaining block is set to be a copy of the leftp for the target block and the rightp for the remaining block is set to be a copy of the rightp for the target block. Next the leftp of the block whose memory address is contained in the rightp of the target block is set to be the memory address of the remaining block and the rightp of the block whose

memory address is held in the leftp of the target  
block is set to be the memory address of the  
remaining block the memory address of the newly  
allocated block is returned as the result of the  
5 "ccmalloc" method

- if the target block is not a wilderness block then the  
address of the target block is passed (together with  
the requested size) as an argument to the method  
"ccdelete"
- 10 • the memory address of the newly allocated block is  
returned as the result of the "ccmalloc" method

#### 4. The method "ccrealloc"

This method resizes an already-allocated (live) block.

- 15 It takes two arguments: the memory address of the block  
to be resized, and the new size that the block should  
have. It returns the address of the resultant block.

- 20 • if the memory address is 0, there is no block to  
resize and so ccrealloc simply passes control to  
the method ccmalloc(size) and returns whatever  
memory address is returned by ccmalloc.
- 25 • if the size is zero, the live block must be freed  
and so ccrealloc simply passes control to the method  
ccfree(address) and then returns a memory address of  
zero.
- 30 • the size of the block at the given memory address  
(the "current" block) is inspected (give it the  
name "current size") and is used to calculate the  
memory address of the next highest adjacent block in  
memory.

- the size of the next highest adjacent block in memory is inspected (we give this the name "nextsize")
- if nextsize indicates that the next block is a wilderness block then the true next size is read from the user data area of the wilderness block
- nextsize is set to its absolute value (the sign bit is ignored)
- if current size is the same as size then no work need be done - the ccrealloc method terminates and returns the original memory address as its result
- if the requested size is greater than the current size AND the current block is not the highest block in memory AND the next highest block in memory is free AND the requested size is less than or equal to MAXFIELDS AND (nextsize + current size + the size of the administration data area) is greater than the requested size AND (nextsize + current size) is greater than (the requested size + the minimum size of the user data area (in this example 8)) THEN perform the following actions:
  - calculate (requested size - current size - size of administration data area) and give it the name "growsize"
  - if growsize is negative, set growsize to zero
  - calculate (current size + growsize + size of administration data area) and give it the name "newsized"
  - pass control to the method "ccdelete", passing it three arguments: (i) the memory address of the next highest block, (ii) growsize and (iii) the Boolean value TRUE (which indicates that a value

for growsize of less than 8 should not be treated as an error nor be rounded up to 8)

- if newsize is greater than MAXFIELDS then terminate with an error message
- 5        • update the administrative data area for the current block such that "my size and availability" is set to be newsize (which should be positive, because this remains a live block)
- 10      • update the administrative data area of the newly-formed next highest block in memory such that its "previous size and availability" is set to be newsize (which should be positive)
- 15      • terminate this method and return the memory address of the current block
- 15      • if the previous test failed, then it is not possible (or not necessary) simply to grow the current block' into the next highest block, so either the block should be shrunk or a new block should be allocated of the required size and the live data in the user data 20 area of the current block should be copied to the new location. In fact, the latter approach subsumes the former in terms of correctness (if not in terms of efficiency!) and so to simplify this example we shall always allocate and copy even in situations where the 25 current block could have been shrunk:

- pass control to the method ccmalloc, passing it the requested size as an argument. The memory address returned by ccmalloc is given the name "cp2"
- 30      • loop N times, where N is the smaller of the requested size and the current size, each time around the loop copying a byte of user data from

the current block to the block at memory address cp2

- pass control to the method ccfree, passing the memory address of the current block as an argument
- 5 terminate this method and return as a result the memory address cp2

#### 5. The method "ccfree"

10 The method ccfree frees a block that is not currently linked into the double-linked chain of free blocks; it does this by linking the block into the double-linked chain of free blocks and updating the pgarray and bitmaps accordingly. It takes as an argument the memory address  
15 of a block to be freed. it does not return any result.

- the memory address passed to ccfree is the address of a block to which we shall give the name "current block"
- the "previous size and availability" for the current  
20 block is inspected and stored in a local variable with the name "prevsize" and the "my size and availability" of the current block is inspected and stored in a local variable with the name "thissize"
- if thissize is -32,768 then read the actual size of the  
25 block from the memory location just after rightp, store this actual value in a local variable with the name "size", and calculate the memory address of the next highest block by adding the value of size (plus the size of the administration data area) to the memory  
30 address of the current block and store this value in a local variable with the name "nextblock"
- if thissize is not -32,768 then calculate the memory address of the next highest block by adding the value of thissize (plus the size of the administration data

area) to the memory address of the current block and store this value in a local variable with the name "nextblock"; also set the value of size to be the same as the value of thissize

- 5 • if prevsize is -32768 then read the actual size of the previous block from the 4 bytes just before the administration data area of the current block (these are the top 4 bytes of the user data area of the previous block) and calculate the memory address of the previous block by subtracting this actual size (together with the size of the administration data area) from the memory address of the current block and store this value in a local variable with the name "prevblock"
- 10 • if prevsize is negative but not -32,768, calculate the memory address of the previous block by adding prevsize to the memory address of the current block and then subtracting the size of the administration data area and store this value in a local variable with the name "prevblock"
- 15 • otherwise, prevsize must be positive, so calculate the memory address of the previous block by subtracting prevsize (plus the size of the administration data area) from the memory address for the current block and store this value in a local variable with the name "prevblock"
- 20 • inspect the "my size and availability" of the next highest block and store it in a local variable with the name "nextsize"
- 25 • 30 • if (thissize is negative) AND (thissize is not -32,768) then terminate the method with an error message (because the only free blocks that might not be already attached to the free list are wilderness blocks)

The ccfree method then determines whether the current block can be coalesced with the next block in memory and/or the previous block in memory (one or both are 5 already free). First the next block is considered, and then the previous block.

- if nextsize is negative then merge this block with the next block by executing the following instructions:
  - 10 • if nextsize is NOT -32,768 then it is not a wilderness block and so execute the following instructions:
    - if (size - nextsize + size of administration data area) is greater than MAXFIELDS then the 15 result of coalescing will be a wilderness block so execute the following instructions:
      - pass control to the ccdelete method, passing three arguments: (i) nextblock, (ii) nextsize, and (iii) the Boolean value FALSE
    - calculate the memory address of the block after the next block as (nextblock - nextsize + the size of the administration data area) and store it in a local variable with the name "nextnextblock"
  - 20 • update the administration data area of the current block such that "my size and availability" is set to -32,768
  - update the memory address just above rightp for the current block such that it contains the 30 value (size - nextsize + size of administration data area)
  - update the memory address 4 bytes before the administrative data area of the block after the

next block, such that it contains the value  
(size - nextsize + size of administration data  
area)

- if the block after the next block exists (i.e.  
5 if it is a valid memory address) then update  
the administration data area of that block such  
that "previous size and availability" is set to  
be -32,768
- if (size - nextsize + size of administration data  
10 area) is NOT greater than MAXFIELDS then the result  
of coalescing will not be a wilderness block so  
execute the following instructions:
  - update size with new value (size - nextsize +  
size of administration data area)
  - update the administration data area of the  
15 current block with the value -size (NB it must  
be negative)
  - calculate the memory address of the block after  
the next block as (nextblock - nextsize + the  
20 size of the administration data area) and store  
it in a local variable with the name  
"nextnextblock"
  - pass control to the method ccdelete, passing it  
three arguments: (i) nextblock, (ii) the value  
25 -size, and (iii) the Boolean value FALSE
  - if the block after the next block exists (i.e.  
if it is a valid memory address) then update  
the administration data area of that block  
such that "previous size and availability" is  
30 set to be -size
- if nextsize IS -32,768 then it is a wilderness block  
and so execute the following instructions:

- get the actual size of the next block and store it in a local variable to which we give the name "bignextsize"
  - pass control to the method ccdelete with three arguments: (i) nextblock, (ii) bignextsize, (iii) the Boolean value FALSE
  - update the administration data area of the current block such that "my size and availability" is set to -32,768
  - calculate the new size which is (size + bignextsize + the size of the administration data area) and update the local variable size to contain this new value
  - store the new value size in the current block at the memory address just after rightp
  - store the new value size in the memory address obtained by adding size to the memory address of the current block minus 4 bytes
  - calculate the value (bignextsize + nextblock + the size of the administration data area) and store this in the local variable with the name "nextnextblock"
- 25            • update the local variable nextblock to have the value stored in the variable nextnextblock

The ccfree method next considers the possibility of merging with the previous block.

- 30     • if prevsize is negative then merge this block with the previous block by executing the following instructions:

- if prevsize is NOT -32,768 then the previous block is not a wilderness block and the following instructions should be executed:
  - update the variable size so that it has a new value given by the calculation (size - prevsize + size of administration data area)
  - pass control to the method ccdelete, passing the three arguments (i) prevblock, (ii) - prevsize, and (iii) the Boolean value FALSE
  - set the current block to be the memory address of prevblock
  - if size is greater than MAXFIELDS then the coalescing has created a wilderness block and the following instructions should be executed:
    - update the administration data area of the current block such that "my size and availability" is set to -32,768
    - save the value size at the memory address just after rightp in the user data area of the current block
    - save the value size at the memory address given by the calculation (prevblock + size - 4)
    - if nextblock is a valid memory address, update the administration data area of nextblock such that "previous size and availability" is set to -32,768
  - if size is not greater than MAXFIELDS then update the administration data area of the current block such that "my size and availability" is set to the value -size; then (if nextblock is a valid memory address) update the administration data area of the next block such that "previous size and availability" is set to -size

- if prevsize IS -32,768 then the previous block is a wilderness block, so execute the following instructions:
  - 5 • read the data held in the memory address 4 bytes before the administration data area forthe current block and store this in a local variable with the name "bigprevsize"
  - 10 • store in the memory address just after rightp for the previous block the value given by the calculation (size + size of administration data area + bigprevsize)
  - 15 • store the value given by the calculation (size + size of administration data area + bigprevsize) into the memory address given by the calculation (prevblock + size + size of administration data area + bigprevsize - 4).
  - 20 • if nextblock is a valid memory address, update the administration data area of nextblock such that "previous size and availability" is set - 32,768
  - 25 • pass control to the method ccdelete with three arguments as follows: (i) prevblock, (ii) size + bigprevsize + size of administration data area, and (iii) the Boolean value FALSE
  - update the current block so that it refers to the memory address stored in prevblock

The ccfree method has at this stage finished all  
30 coalescing and subsequently adds the current block  
(possibly now coalesced) into the free list and pgarray.

- inspect the administrative data area of the current block and store the value of "my size and availability" into a local variable with the name "thissize"
- 5 • if thissize is -32,768 then the current block is a wilderness block and the following instructions should be executed:
  - the current block is inserted at the top of the double-linked chain of free blocks, pointed to directly by the max block. This is achieved by updating the leftp of the current block to contain the memory address of the max block, by updating the rightp of the current block to contain a copy of the value currently stored in the rightp of the max block, by updating the leftp of the block whose memory address is stored in the rightp of the max block so that it contains the memory address of the current block, and by updating the rightp of the max block so that it contains the memory address of the current block
  - 10 • the ccfree method then terminates and control passes to the method that called ccfree.
- 20 • if thissize is NOT -32,768 then the current block is not a wilderness block and the following instructions should be executed:
  - set thissize to be the absolute value of thissize (i.e. make it positive if it is negative)
  - 25 • if (thissize is less than or equal to MAXFIELDS) AND (pgarray is not empty) then link the current block into the double-linked list of free blocks just above the block whose memory address is held in

pgarray[thissize], and make pgarray[thissize] hold the memory address of the current block.

This is done by executing the following instructions:

- 5           • store in the leftp of the current block the memory address currently stored in the leftp of the block whose memory address is stored in pgarray[thissize]
- 10          • store in the leftp of the block whose memory address is held in pgarray[thissize] the memory address of the current block
- 15          • find the block whose memory address is now stored in the leftp of the current block and in the rightp of that block store the memory address of the current block
- 20          • store in the rightp of the current block the memory address that is held in pgarray[thissize]
- 25          • Note that we do NOT make pgarray[thissize] point to the current block yet - this is done later
- if either (thissize is greater than MAXFIELDS) OR (pgarray[thissize] is empty) then it will be necessary to search for the correct place in the double-linked chain of free blocks in which to insert the current block. This is done by executing the following instructions:
  - 30           • pass control to the method "set\_and\_search\_bitmap" with one argument passed, which is thissize; this method will return a number which is then stored in a local variable with the name "i"

- update the rightp for the current block so that it contains the memory address held in pgarray[i]
- update the leftp for the current block so that it contains the data in the leftp of the block whose memory address is held in pgarray[i]
- update the leftp of the block whose memory address is held in pgarray[i] so that it contains the memory address of the current block
- update the rightp of the block whose memory address is held in the leftp of the current block so that it contains the memory address of the current block
- Next update the administration data area of the current block so that "my size and availability" is -thissize
- If the next block is a valid memory address, update its administration data area such that "previous size and availability" is -thissize
- If thissize is less than or equal to MAXFIELDS, update the contents of pgarray[thissize] so that it contains the memory address of the current block

## 6. the subsidiary method "ccdelete"

The method ccdelete is used internally (by ccmalloc and 30 ccfree) - it deletes (unlinks) a specified block from the double-linked chain of free blocks and removes any reference to that block from either the pgarray or the bitmaps. It will also manage the splitting of such a

- deleted block if required, with the lower sub-sub-block remaining deleted and the upper sub-block being reinserted back into the double-linked list of free blocks (and pgarray and bitmaps updated accordingly).
- 5   The method takes three arguments (i) the memory address of a block to be deleted, which will be stored in a local variable with the name "cp" (ii) the size of the block to be deleted, which will be stored in a local variable with the name "size" (if it is less than the actual size of
- 10   the block, this indicates that a split is requested), and (iii) a Boolean value to indicate whether it is OK to split just a very small (< 8 bytes) block from the start of the given free block
- this will be stored in a local variable with the name
- 15   "small\_is\_ok".
- inspect the administration data area of the block whose memory address is held in cp (we give this the name "current block") and store the "my size and availability" data into a new local variable with the
- 20   name "mysize"
- if mysize is negative and mysize is not -32,768 then update mysize with the value -mysize
  - if size is negative then update size with the value - size
- 25   • read the data held in rightp of the current block and save it in a new local variable with the name "rightblock"
- store the value held in rightblock into the rightp of the block whose memory address is held in the leftp of the current block
- 30   • read the value stored in the leftp of the current block and store it into the leftp of the block whose memory address is held in rightblock

- if (mysize is not -32,78) AND (pgarray[mysize] is equal to cp) then execute the following instructions:
  - if the "my size and availability" data in the administration data area of the block whose address is stored in rightblock is equal to - mysize then update pgarray[mysize ] so that it holds the memory address that is currently stored in rightblock
  - else update pgarray[mysize] to contain an empty address (i.e. the value zero) and then pass control to the method "clear\_bitmap", with mysize as an argument (and, when the method clear\_bitmap returns, continue executing the next instruction for this method)
- 15 • if mysize is not -32,768 then (i) update the administration data area of the current block such that "my size and availability" is set to the value mysize, and (ii) if the next highest block (given by the memory address cp + mysize + size of 20 adminstration data area) is a valid memory address, update the "previous size and availability" data of the administration data area of the next block to be the value mysize
- if (size is less than 8) AND (small\_is\_ok is FALSE)
  - 25 then set size to be 8
  - if mysize is -32,768 then read the actual size from the memory address just after rightp in the user data area of the current block and store it in a new local variable with name "bigmysize"
- 30 • else store the value currently held in mysize into a new local variable with name "bigmysize"

- if (bigmysize - size - the size of the administration data area) is less than 8 then update size to be the value held in bigmysize
- if size is bigger than bigmysize then terminate this 5 method with an error message
- if bigmysize is bigger than size then execute the following instructions:
  - calculate (cp + size) and store the result in a new local variable with the name "cp2"
  - 10 • if ((bigmysize - size - the size of the administration data area) is greater than MAXFIELDS) then execute the following instructions:
    - update the administration data area of the 15 block whose memory address is held in cp2 such that "my size and availability" is set to the value -32,768
    - if (cp + bigmysize + size of the administration data area) is a valid memory address then update the administration data area of the block at that address such that 20 "previous size and availability" is set to -32,768
    - store the value (bigmysize - size - the size 25 of the administration data area) into the memory location given by (cp2 + 8 + size of administration data area)
    - store the value (bigmysize - size - the size 30 of the administration data area) into the memory location given by (cp2 + bigmysize - 4)
  - else execute the following instructions:

- update the administration data area of the block whose memory address is held in cp2 such that "my size and availability" is set to the value (bigmysize - size - size of administration data area)  
5
- if (cp + bigmysize + size of the administration data area) is a valid memory address then update the administration data area of the block at that address such that "previous size and availability" is set to (bigmysize - size - size of administration data area)  
10
- update the administration data area of the block whose address is stored in cp2 such that "previous size and availability" is set to the value held in size  
15
- update the administration data area of the current block such that "my size and availability" is set to the value held in size  
20
- pass control to the method "ccfree", with the memory address currently held in cp2 as an argument  
25
- else if (cp + bigmysize + size of administration data area) is a valid memory address then execute the following instructions:
  - calculate (cp + bigmysize + size of data administration area) and store the result in a new local variable with name "nextblock"  
30
  - if bigmysize is greater than MAXFIELDS then update the administration data area ofnextblock such that "previous size and availability" is set to -32,768

- else update the administration data area of nextblock such that "previous size and availability" is set to bigmysize

5 7. The subsidiary method "search"

This method takes a single argument "size" and searches the bitmaps to find the highest set bit that is less than the bit representing the pgarray index "size". It  
10 returns the appropriate index.

- if size is greater than MAXFIELDS set size to be MAXFIELDS
- calculate the integer result of dividing size by 32 and  
15 store this value in a local variable with the name "bitindex"
- calculate the remainder after dividing size by 32 and store this value in a local variable with the name "bitremainder"
- 20 • if bitremainder is zero execute the following instructions:
  - if bitindex is zero then terminate this method and return the value zero
  - if bitindex is not zero then update bitindex with  
25 the result of subtracting one from the existing value of bitindex and then update bitremainder so that it contains the value 32
- create a 32 bit integer with only the top bit set and all other bits being zero (i.e. 0x80000000) and store  
30 it in a local variable with name "msb"
- calculate the result of right-shifting msb (bitremainder-1) times and store the result in a local

variable with the name "mask". The value stored in msb should NOT be altered.

- store the value  $(\text{bitremainder}-1)$  in a local variable with the name "i"

5 • loop until i is no longer positive, executing the following instructions each time around the loop:

- if the result of the bitwise AND of bitmap0[bitindex] with mask is not zero then terminate this method and return the value given by the calculation  $((\text{bitindex} \times 32) + i)$
- update mask to be the result of the existing value of mask left-shifted once
- update i to contain  $(i - 1)$

15

- calculate the integer result of dividing  $(\text{bitindex} - 1)$  by 32 and store this value in a local variable with the name "j"

- calculate the remainder after dividing  $(\text{bitindex} - 1)$

20 by 32 and store this value in a local variable with the name "searchtop"

- update mask to contain the result of right-shifting msb searchtop times (the value held in msb should not be altered)

25 • set a local variable "foundit" to be the Boolean value FALSE

- update i to contain the value currently held in searchtop

- loop until i is no longer positive, executing the

30 following instructions each time around the loop:

- if the result of the bitwise AND of bitmap1[j] with mask is not zero then update foundit to contain the Boolean value TRUE and store the

- value currently held in j into a new local variable with name "bitindex2" and store the value currently held in i into a new local variable with the name bitindex1 and then break  
5 out of this loop and continue with the remaining instructions for this method
- update mask to be the result of the existing value of mask left-shifted once
  - update i to contain (i - 1)
- 10 • if foundit contains the Boolean value FALSE then execute the following instructions:
- update mask to contain the result of right-shifting msb a number of times that is given by the remainder after dividing (j-1) by 32
  - update i to contain the remainder after dividing (j-1) by 32
  - loop until i is no longer positive, executing the following instructions each time around the loop:
    - if the result of the bitwise AND of bitmap2[0] with mask is not zero, update foundit to contain the Boolean value TRUE and update bitindex2 to contain the value currently held in i and then break out of this loop and continue with the remaining instructions for this method
    - update mask to be the result of the existing value of mask left-shifted once
    - update i to contain (i - 1)
  - if foundit contains the Boolean value FALSE then there has been an internal error and the method should terminate with an error message

- update mask to contain the result of right-shifting msb 31 times. The value held in msb should not be altered.
- update foundit to contain the Boolean value FALSE
- 5 • update i to contain the value 31
- loop until i is no longer positive, executing the following instructions each time around the loop:
  - if the result of the bitwise AND of bitmap1[bitindex2] with mask is not zero then update foundit to contain the Boolean value TRUE and update bitindex1 with the value currently held in i and then break out of this loop and continue with the remaining instructions for this method
- 10 • update mask to be the result of the existing value of mask left-shifted once
- update i to contain (i - 1)
- if foundit contains the Boolean value FALSE then there has been an internal error and the method should terminate with an error message
- 20
- update mask to contain the result of right-shifting msb 31 times. The value held in msb shold not be altered.
- update foundit to contain the Boolean value FALSE
- 25 • update j to contain the value 31
- loop until j is no longer positive, executing the following instructions each time around the loop:
  - if the result of the bitwise AND of bitmap0[bitindex1 + (bitindex2 \* 32)] with mask is not zero then update foundit to contain the Boolean value TRUE and update bitremainder with the value currently held in j and then break out of this loop
- 30

and continue with the remaining instructions for this method

- update mask to be the result of the existing value of mask left-shifted once
- 5     • update j to contain (j - 1)
- if foundit contains the Boolean value FALSE then there has been an internal error and the method should terminate with an error message
- terminate this method and return as the result the
- 10    value obtained by the calculation ((bitindex1 + (ibtindex2 \* 32)) \* 32) + bitremainder)

#### 8. The subsidiary method "clear\_bitmap"

- 15 This method takes a single argument "size" and updates the bitmaps so that they reflect the fact that there is no valid memory address in pgarray[size]. There is no return value.
  - create a 32 bit integer with only the top bit set and all other bits being zero (i.e. 0x80000000) and store it in a local variable with name "msb"
  - calculate the integer value of (size / 32) and store the result in a local variable with the name "bitindex"
- 20
  - calculate the remainder after size has been divided by 32 and store the result in a local variable with the name "bitremainder"
  - update bitmap0[bitindex] with the result of calculating the bitwise AND of the current value stored in bitmap0[bitindex] and the result of (right-shifting msb bitremainder times and then inverting every bit). The value of msb should not be altered.
- 25
  - if bitmap0[bitindex] is zero then :
- 30
  -

- update bitremainder to be the remainder after bitindex has been divided by 32
- update bitindex to be the integer result of dividing the existing value held in bitindex by 32
- 5 • update bitmap1[bitindex] with the result of calculating the bitwise AND of the existing value of bitmap1[bitindex] with the result of (right-shifting msb bitremainder times and then inverting all the bits). The value of msb should not be altered.
- 10 • else:
  - update bitindex to be the integer result of dividing the existing value held in bitindex by 32
  - if bitmap1[bitindex] is zero then:
  - 15 • update bitremainder to be the remainder after bitindex has been divided by 32
  - update bitindex to be the integer result of dividing the existing value held in bitindex by 32
  - update bitmap2[bitindex] with the result of calculating the bitwise AND of the existing value of bitmap2[bitindex] with the result of (right-shifting msb bitremainder times and then inverting all the bits).
- 20 25 9. The subsidiary method "set\_and\_search\_bitmap" This method achieves two objectives: it updates the bitmaps to indicate that pgarray[size] contains a valid memory address, and it searches for the biggest set bit that represents a pgarray index less than size. It then returns the appropriate index.
- 30

- pass control to method "search", passing the argument size; the method "search" will return an integer that is then stored in a local variable with the name "i"
- if size is less than or equal to MAXFIELDS, then update 5 the bitmaps as follows:
  - create a 32 bit integer with only the top bit set and all other bits being zero (i.e. 0x80000000) and store it in a local variable with name "msb"
  - calculate the integer value of (size / 32) and 10 store it in a local variable with name "bitindex"
  - calculate the remainder after size is divided by 32 and store that remainder in a local variable with name "bitremainder"
  - update bitmap0[bitindex] with the result of calculating the bitwise OR of the existing value of bitmap0[bitindex] with the value obtained by right-shifting the value stored in msb bitremainder times (the value held in msb should NOT be altered)
  - update bitmap1[bitindex / 32] with the result of calculating the bitwise OR of the existing value of bitmap1[bitindex/32] with the value obtained by right-shifting msb by the number of times that is the result of calculating the remainder after dividing bitindex by 32 (the value held in msb should NOT be altered)
  - update bitmap2[(bitindex/32)/32] with the result of calculating the bitwise OR of the existing value of bitmap2[(bitindex/32)/32] with the value obtained by right-shifting msb by the number of times that is the result of calculating the remainder after (bitindex/32) is divided by 32

- terminate this method and return the value i

Claims

1. A data structure comprising:
  - 5 a data storage space;
  - a first data array which, for each of one or more-cell sizes, stores data indicating whether or not the data storage space includes at least one cell of a certain type and of the respective size; and
  - 10 a second data array which, for any said size for which at least one cell of the determined type exists, indicates a location of at least one cell of that size and type within the data storage space.
- 15 2. A data structure according to claim 1 in which said predetermined cell sizes are no more than a predetermined maximum cell size which is independent of the total size of the data storage space.
- 20 3. A data structure according to claim 2 in which said predetermined cell sizes are all cell sizes up to the predetermined maximum cell size (MAX FIELDS).
- 25 4. A data structure according to claim 3 in which the first data array is selected so that for a cell size n it is possible to determine the smallest (integer) value of p such that p>n and there exists at least one cell of the predetermined type of size p, within a time which scales with log (MAXFIELDS).
- 30 5. A data structure according to claim 3 in which the first data array is selected so that for a cell size n it is possible to determine the largest (integer) value of p

such that  $p < n$  and there exists at least one cell of the predetermined type of size  $p$ , within a time which scales with  $\log(\text{MAXFIELDS})$ .

5 6. A data structure according to any preceding claim in which the first *data array* is a hierarchical *data array*, having:

a first level which includes one or more first fields, each first field indicating for a respective size 10 of cell the existence or otherwise of at least one cell of that size, said first fields being divided into one or more groups, each group representing a respective range of cell sizes; and

15 a second level which includes one or more second fields, each second field indicating for a respective said group the existence or otherwise of at least one cell in the respective range of cell sizes.

20 7. A data structure according to claim 6 in which the first *data array* contains further layers above the second layer, there being in each  $i$ -th layer one or more fields, for each layer except the top layer, the one or more fields are divided into one or more groups, each group representing a range of cell sizes, and in the  $(i+1)$ -th layer there is a field for each respective group of the 25  $i$ -th layer.

8. A data structure according to claim 7 in which in each layer, each group represents a number of fields in the layer below which is a multiple of eight.

30 9. A data structure according to any of claims 6 to 8 in which each of said fields of at least the first level indicates the existence or otherwise of at least one cell of said size or size range by a binary variable.

10. A data structure comprising:

- a *data storage space*;
- a first *data array* which, for each of one or more *cell size ranges*, stores data indicating whether or not the *data storage space* includes
- 5 at least one *cell* of a certain type within the respective *size range*; and
- a second *data array* which, for any of said *size ranges* for which at least one *cell* of the determined type exists, indicates a location of at
- 10 least one of those *cells* within the *data storage space*.
11. A data structure according to any preceding claim which includes a *field* which stores the location of at least one of: (i) a *cell* of the predetermined type having
- 15 the greatest *size* for which *cells* of the predetermined type exist, or (ii) a *cell* of the predetermined type having the smallest *size* for which *cells* of the predetermined type exist.
- 20 12. A data structure according to any preceding claim in which the *cells* contain pointers to each other such that, starting from any *cell* of any predetermined *cell size*, any other *cell* of the predetermined type and the same *cell size* can be reached by moving successively from one
- 25 *cell* to another according to the pointers.
13. A data structure according to claim 12 in which at least one *cell* of each predetermined *size* contains a pointer to one of the *cells* of the next higher occupied
- 30 predetermined *cell size*, and/or at least one *cell* of each predetermined *size* contains a pointer to one of the *cells* of the next lower occupied predetermined *size*.

14. A data structure according to claim 12 or claim 13 in which the order of the path defined by the pointers through *cells* of the same *size* encodes information about the *cells* of that *size*.

5

15. A data structure according to claim 12, claim 13 or claim 14 which includes at least one *cell* of the predetermined type which is of minimum *size*, has a predetermined location in the *data storage space* and 10 contains a pointer to a *cell* of the predetermined type of the same or next higher *size*.

16. A data structure including:

a *data storage space*, the *data storage space* 15 comprising one or more *cells* of a certain type, each said *cell* of said type containing at least one pointer to the, location in the *data storage space* of another said *cell* of said type; and

a *field* which stores as one or more pointers the 20 location within the *data storage space* of at least one of: (i) one of said *cells* of said type which is at least as large as any other of said *cells* and of said type, or (ii) one of said *cells* of said type which is at 25 least as small as any other of said *cells* and of said type;

said pointers being such that, starting from a *cell* of which the location is indicated in said *field*, any other of said *cells* can be reached by moving successively 30 from one *cell* to another according to the pointers in a path in which the *size* of the *cells* varies monotonically.

17. A memory management method for controlling one or more *data storage devices* which support a *data storage space*, the memory management method including:

5 deriving a first *data array* which, for each of one or more *cell sizes*, stores data indicating whether or not the *data storage space* includes at least one free cell of the respective size;

10 deriving a second *data array* which, for any said size for which at least one free cell exists, indicates a location of at least one free cell of that size within the *data storage space*; and

15 upon receiving an instruction to store a *data item*, determining using said first *data array* an appropriate *cell size* of a free cell for storing the *data item*, and using said second *data array* to derive the location within the *data storage space* of a location of a free cell of the determined *cell size*.

18. A memory management method for controlling one or 20 more *data storage devices* which support a *data storage space*, the *data storage space* containing:

25 one or more *free cells*, each said cell containing at least one pointer to the location in the *data storage space* of another said cell; and

30 a *field* which stores the location within the *data space* of at least one of: (i) one of said cells which is at least as large as any other of said cells, or (ii) one of said cells which is at least as small as any other of said cells;

35 said pointers being such that, starting from a *cell* of which the location is indicated in said *field*, any other of said *cells* can be reached by moving successively from one *cell* to another according to

the pointers in a path in which the size of the cells varies monotonically;

the method including identifying a free cell of a certain size by starting from a first free cell, and

- 5 moving successively from one cell to another according to the pointers until a cell of that size is reached.

19. A memory management method for controlling one more

data storage devices which support a data storage space

- 10 containing one or more zombie cells, each zombie cell containing at least one pointer to the location in the data storage space of another zombie cell;

the method employing afield which stores the location within the data storage space of one of said

- 15 zombie cells which is at least as large, or at least as

sl:llall, as any other of said zombie cells;

said pointers being such that, starting from a zombie cell of which the location is indicated in said field, any other of said zombie cells can be reached by

- 20 moving successively from one zombie cell to another according to the pointers in a path in which the size of the zombie cells varies monotonically;

the method including freeing zombie cells in an order defined by starting from a first zombie cell and

- 25 moving successively from one zombie cell to another according to the pointers.

20. A memory management method for controlling one more

data storage devices which support a data storage space containing one or more cells, each cell containing at

- 30 least one pointer to the location in the data storage space of another cell, and each cell additionally being associated with at least one integer property;

the method employing afield which stores the

location within the data storage space of one of said

cells which contains a value of said property which is at least as large, or at least as small, as any other value of said property of said cells;

said pointers being such that, starting from a cell 5 of which the location is indicated in said field, any other of said cells can be reached by moving successively from one cell to another according to the pointers in a path in which the value of said property of the cells varies monotonically;

10 the method including deleting cells in an order defined by starting from a first cell, and moving successively from one cell to another according to the pointers.

15 21. A data storage device which is arranged to store a data structure according to any of claims 1 to 16.

22. A data storage device which performs a method according to any of claims 17 to 20.

20 23. A computer system for performing a computational task comprising a data storage device according to claim 21 or 22.

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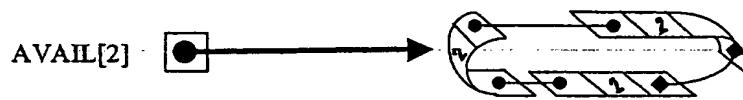
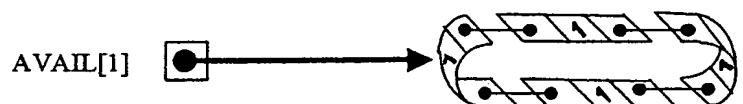
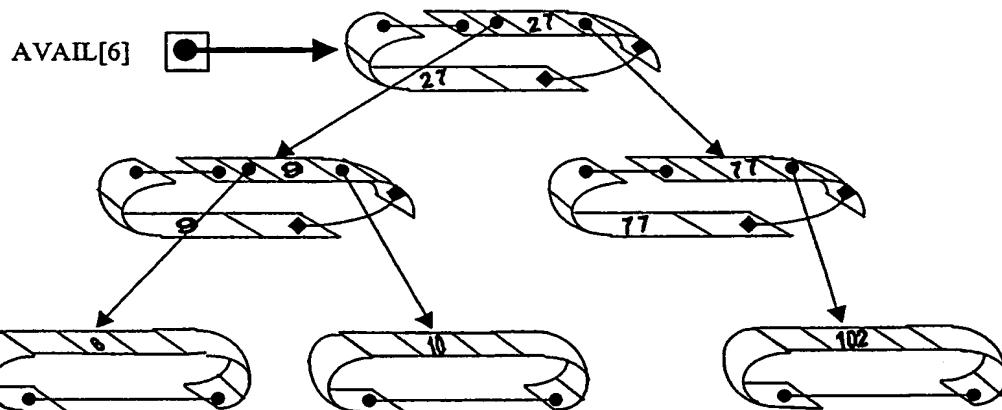
AVAIL[3] 

Fig. 1

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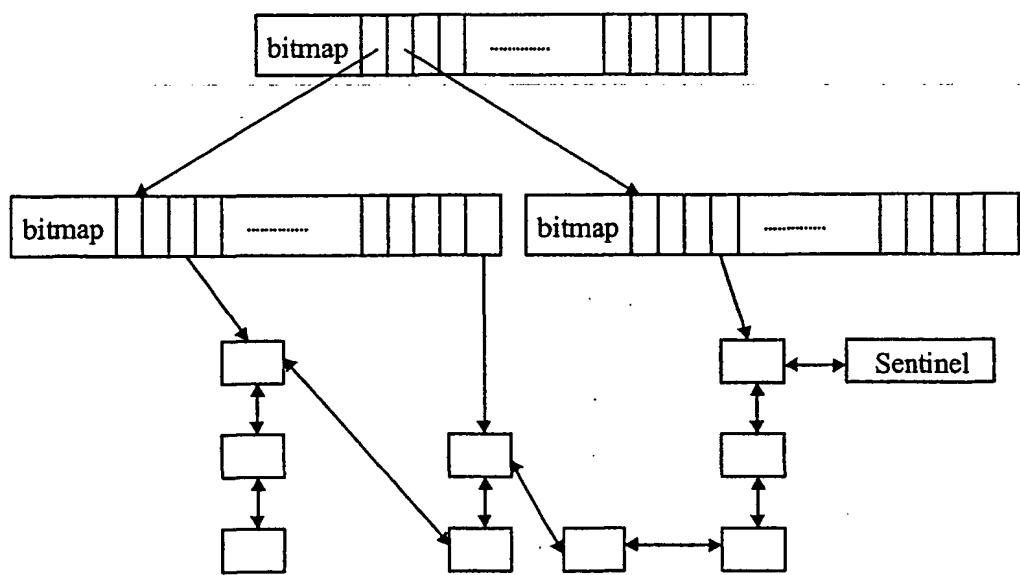


Figure 2

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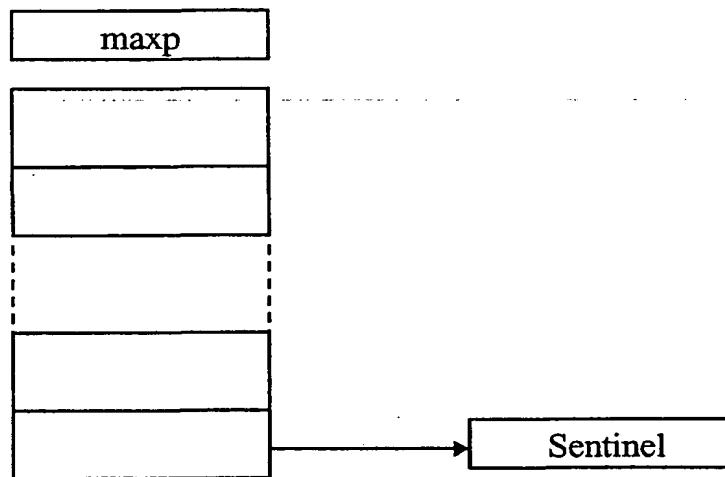


Figure 3 : Initial State of a Peer-Group List

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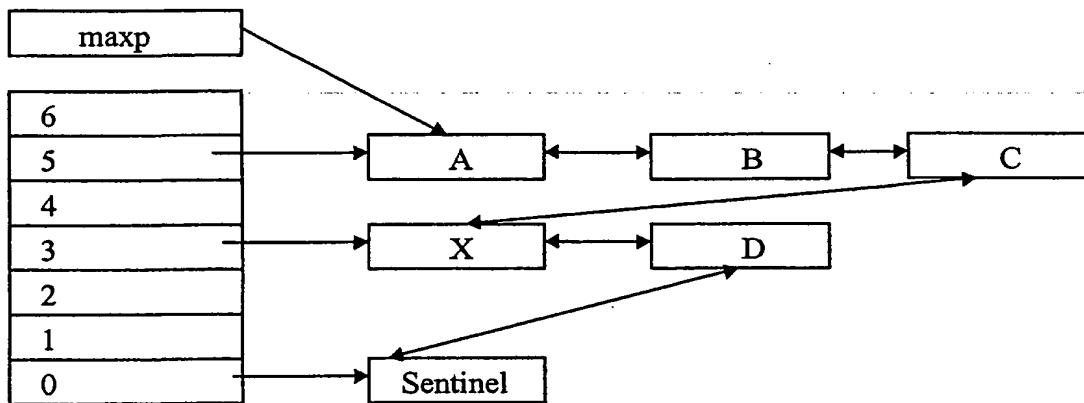


Figure 4 : Before Deletion of cell X

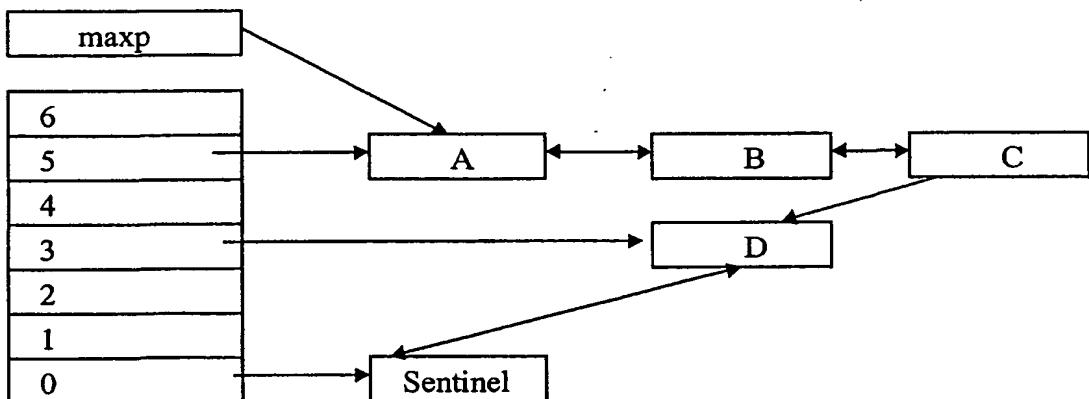


Figure 5 : After Deletion of cell X

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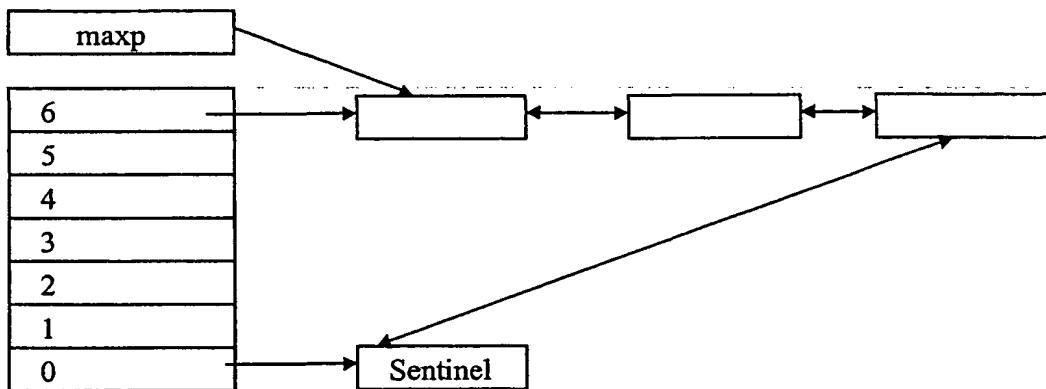


Figure 6 : Before Insertion of cell

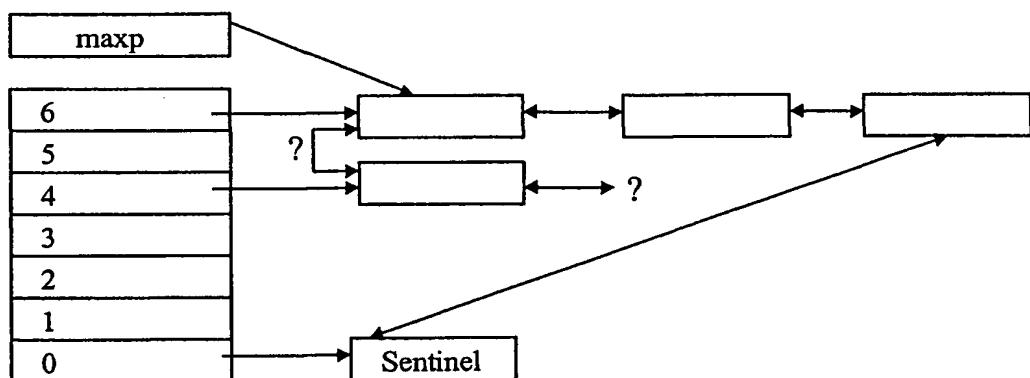


Figure 7 : A problem with Insertion

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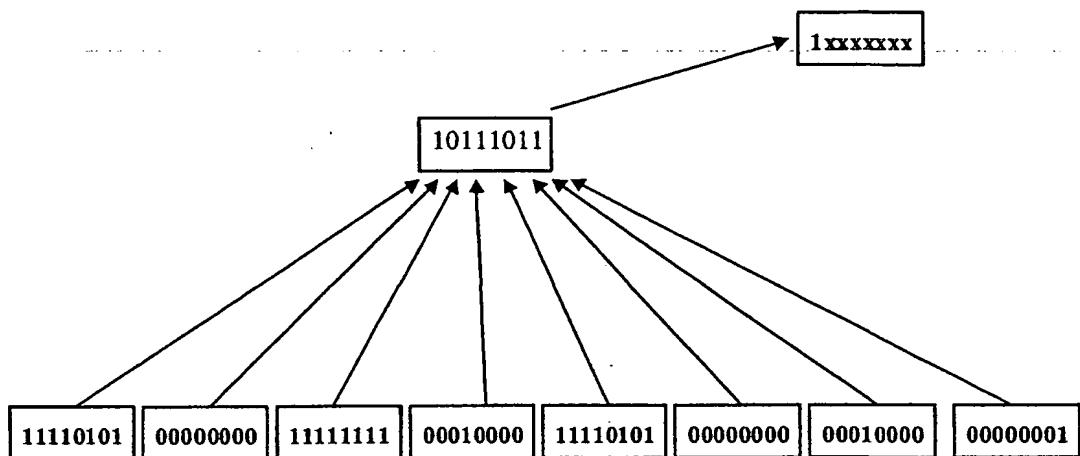


Figure 8 : Bitmap Abstractions

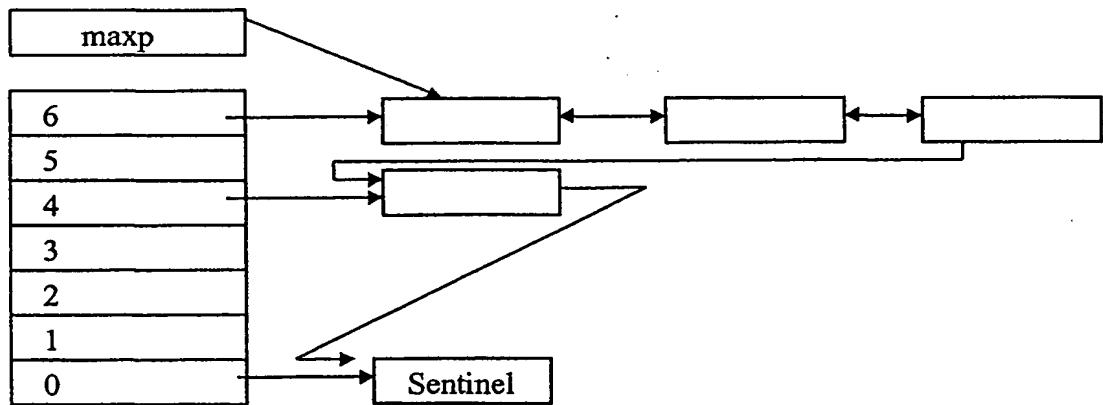


Figure 9 : Insertion problem resolved

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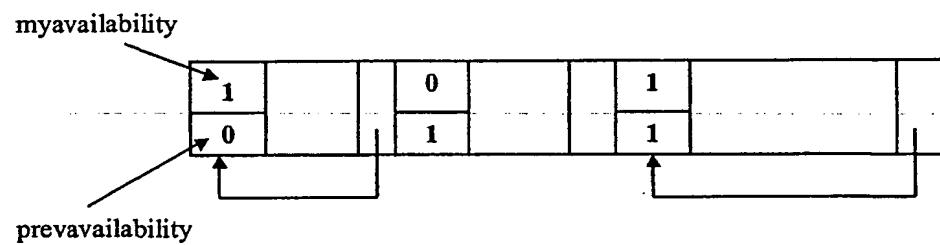


Figure 10 : Availability of Cells

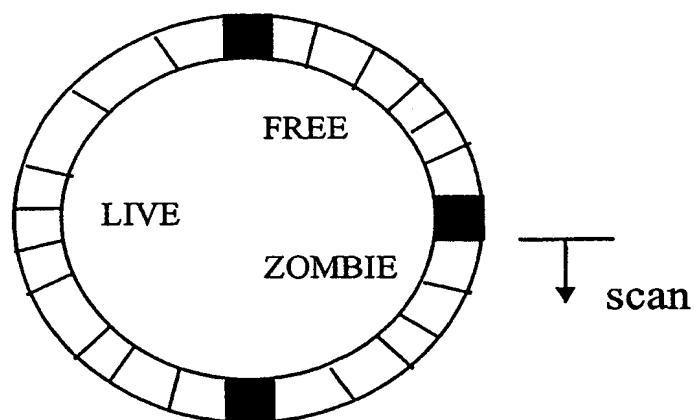


Figure 11: Treadmill Similarity